Homework 13

section 7.2

18) Let $f(x,y,z)=\frac{xy}{z}$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$

$f(x,y,z)=\frac{xy}{z}=xyz^{-1}$ *

$\frac{\partial f}{\partial x}=yz^{-1}$

$\frac{\partial f}{\partial y}=xz^{-1}$

$\frac{\partial f}{\partial z}=xy(-z^{-2})$

24) Let $f(x,y)=xe^y+x^4y+y^3$. Find $\partial^2 f/\partial x^2$, $\partial^2 f/\partial y^2$, $\partial^2 f/\partial x\partial y$, and $\partial^2 f/\partial y\partial x$

$f(x,y)=xe^y+x^4y+y^3$

$\frac{\partial f}{\partial x}=e^y+4x^3y+0=e^y+4x^3y$, $\frac{\partial f}{\partial y}=xe^y+x^4+3y^2$

$\Rightarrow \frac{\partial^2 f}{\partial x^2}=0+12x^2y=12x^2y$, $\frac{\partial^2 f}{\partial y^2}=xe^y+0+6y=xe^y+6y$

$\frac{\partial^2 f}{\partial x\partial y}=e^y+4x^3+0=e^y+4x^3$, $\frac{\partial^2 f}{\partial y\partial y}=e^y+4x^3$

Section 7.3

18) The function $f(x,y)=\frac{1}{2}x^2+2xy+3y^2-x+2y$ has a minimum at some point (x,y). Find the values of x and y where this minimum occurs *

$f(x,y)=\frac{1}{2}x^2+2xy+3y^2-x+2y$

$\frac{\partial f}{\partial x}=x+2y+0-1+0=x+2y-1$

$\frac{\partial f}{\partial y}=0+2x+6y-0+2=2x+6y+2$

$x+2y-1=0$ $\Rightarrow$ $x+2y-1=0$ $\overset{\text{subtracting}}{\Rightarrow}$ $-y-2=0 \Rightarrow y=-2 \Rightarrow x-4-1=0 \Rightarrow x=5$

$2x+6y+2=0$ $\quad$ $x+3y+1=0$

$\Rightarrow \boxed{(5,-2)}$

20) Both first partial derivatives are zero at the given points. Use the second derivative test to determine the nature of f(x,y) at each of these points:

$f(x,y) = 6xy^2 - 2x^3 - 3y^4$; (0,0), (1,1), (1,-1)

*

$\frac{\partial f}{\partial x} = 6y^2 - 6x^2$

$\frac{\partial f}{\partial y} = 6x(2y) - 12y^3 = 12xy - 12y^3$

$\frac{\partial^2 f}{\partial x^2} = -12x \quad \frac{\partial^2 f}{\partial y^2} = 12x - 36y^2 \quad \frac{\partial^2 f}{\partial x \partial y} = 12y$

$D(x,y) = (-12x)(12x - 36y^2) - (12y)^2$

$D(0,0) = 0 - 0 = 0$ inconclusive

$D(1,1) = (-12)(12-36) - (12)^2 = 288 - 144 = 144 > 0$, $\frac{\partial^2 f}{\partial^2 x}(1,1) = -12 < 0$ max

$D(1,-1) = (-12)(12-36) - (-12)^2 = 288 - 144 = 144 > 0$ $\frac{\partial^2 f}{\partial x^2}(1,-1) = -12 < 0$ max

30) Find all points (x,y) where f(x,y) has a possible relative maximum or minimum. Then, use the second-derivative test to determine, if possible, the nature of f(x,y) at each of these points.

$f(x,y) = x^2 + 2xy + 10y^2$

$\frac{\partial f}{\partial x} = 2x + 2y$

$\frac{\partial f}{\partial y} = 2x + 20y$

$2x + 2y = 0 \xRightarrow{\text{subtract}} -18y = 0 \Rightarrow y = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$

$2x + 20y = 0$

(0,0) critical point

$\frac{\partial^2 f}{\partial x^2} = 2 \quad \frac{\partial^2 f}{\partial y^2} = 20 \quad \frac{\partial^2 f}{\partial x \partial y} = 2$

$D(x,y) = 2(20) - (2)^2 = 40 - 4 = 36 > 0$

$\frac{\partial^2 f}{\partial x^2} = 2 > 0$ min

CW Graded: CW 21 #1c, #2b ; CW22 #1