Homework 2

section 1.4

10) Determine if the limit exists. Compute the limit if it exists: $\lim_{x\to 4}(x^3-7)$

$\lim_{x\to 4}(x^3-7) = (4)^3-7 = 64-7 = \boxed{57}$

22) Determine if the limit exists. Compute the limit if it exists: $\lim_{x\to 5}\frac{2x-10}{x^2-25}$

$\lim_{x\to 5}\frac{2x-10}{x^2-25}$ (Try to plug in: $\frac{2(5)-10}{(5)^2-25} = \frac{10-10}{25-25} = \frac{0}{0}$ Didn't work! Try something else!)

$= \lim_{x\to 5}\frac{2\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \lim_{x\to 5}\frac{2}{x+5} = \frac{2}{5+5} = \frac{2}{10} = \boxed{\frac{1}{5}}$

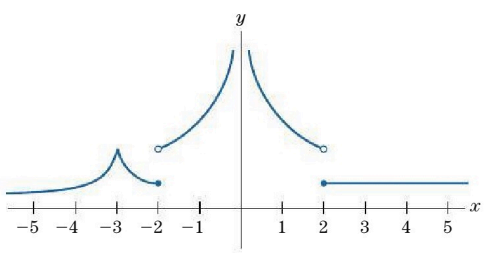
*

70) Find $\lim_{x\to\infty}\frac{-8x^2+1}{x^2+1}$

$\lim_{x\to\infty}\frac{-8x^2+1}{x^2+1} = \lim_{x\to\infty}\frac{-8x^2/x^2+1/x^2}{x^2/x^2+1/x^2} = \lim_{x\to\infty}\frac{-8+\frac{1}{x^2}}{1+\frac{1}{x^2}} = \frac{-8+0}{1+0} = \boxed{-8}$

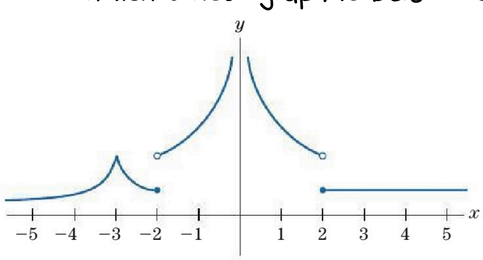
section 1.5

2) Is the function whose graph is below continuous at the following value of a: a=-3



yes

6) Is the function whose graph is below continuous at the following value of a: x=2



no

16) Determine whether the following function is continuous and differentiable at $x=1$:

$$f(x) = \begin{cases} x^3, & 0 \le x < 1 \\ x, & 1 \le x \le 2 \end{cases}$$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^3 = (1)^3 = 1$$

*** Differentiability not part of assignment**

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x = 1$$

$$f(1) = 1$$

yes continuous at $x=1$

$$f'(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$$

$$\left.\begin{aligned} \lim_{x\to 1^-} f'(x) &= \lim_{x\to 1^-} (3x^2) = 3 \\ \lim_{x\to 1^+} f'(x) &= \lim_{x\to 1^+} 1 = 1 \end{aligned}\right\} \text{not differentiable at } x=1$$

<u>section 1.4</u>

30) Use limits to compute $f'(2)$, where $f(x) = x^3$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^3-(2)^3}{h} = \lim_{h\to 0} \frac{(2+h)(2+h)(2+h)-8}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)(4+4h+h^2)-8}{h} = \lim_{h\to 0} \frac{\cancel{8}+8h+2h^2+4h+4h^2+h^3-\cancel{8}}{h} = \lim_{h\to 0} \frac{h^3+6h^2+12h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h^2+6h+12)}{\cancel{h}} = \lim_{h\to 0} h^2+6h-12 = (0)^2+6(0)+12 = \boxed{12}$$

32) Use limits to compute $f'(0)$, where $f(x) = x^2+2x+2$

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{f(h)-f(0)}{h} = \lim_{h\to 0} \frac{(h^2+2h+2)-(0+0+2)}{h} = \lim_{h\to 0} \frac{h^2+2h+\cancel{2}-\cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h+2)}{\cancel{h}} = \lim_{h\to 0} (h+2) = \boxed{2}$$

37) Use limits to compute $f'(x)$: $f(x) = 3x+1$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[3(x+h)+1]-[3x+1]}{h} = \lim_{h\to 0} \frac{\cancel{3x}+3h+\cancel{1}-\cancel{3x}-\cancel{1}}{h} = \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 3 = \boxed{3}$$

45) Use limits to compute $f'(x)$: $f(x) = \sqrt{x+2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} = \lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$= \lim_{h\to 0} \frac{(x+h+2)-(x+2)}{h(\sqrt{x+h+2}+\sqrt{x+2})} = \lim_{h\to 0} \frac{\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}}{h(\sqrt{x+h+2}+\sqrt{x+2})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$$= \frac{1}{\sqrt{x+2}+\sqrt{x+2}} = \boxed{\frac{1}{2\sqrt{x+2}}}$$

(classwork problems graded: CW 2 #2 and CW3 #1)