

Homework 3

Section 1.6

14) Differentiate $y = 4(x^2 - 6)^{-3}$

$$\frac{dy}{dx} = 4(-3(x^2 - 6)^{-4} \frac{d}{dx}(x^2 - 6)) = 4(-3(x^2 - 6)^{-4}(2x)) = \boxed{-24(x^2 - 6)^{-4}}$$

20) Differentiate $y = (x^2 + 1)^2 + 3(x^2 - 1)^2$

$$\begin{aligned} \frac{dy}{dx} &= 2(x^2 + 1) \frac{d}{dx}(x^2 + 1) + 3(2(x^2 - 1) \frac{d}{dx}(x^2 - 1)) = 2(x^2 + 1)(2x) + 3(2(x^2 - 1)(2x)) \quad * \\ &= \boxed{4x(x^2 + 1) + 12(x^2 - 1)} \end{aligned}$$

32) Differentiate $y = \frac{7}{\sqrt{1+x}}$

$$y = \frac{7}{\sqrt{1+x}} = 7(1+x)^{-1/2} \quad *$$

$$\frac{dy}{dx} = 7\left(-\frac{1}{2}(1+x)^{-3/2} \frac{d}{dx}(1+x)\right) = \boxed{-\frac{7}{2}(1+x)^{-3/2}}$$

42) Write the equation of the tangent line to the curve $y = x^3 + 3x - 8$ at $(2, 6)$

$$\frac{dy}{dx} = 3x^2 + 3 \quad m = 3(2)^2 + 3 = 3(4) + 3 = 15$$

$$\boxed{y - 6 = 15(x - 2)}$$

44) Find the equation of the tangent line to the curve $y = \frac{8}{x^2 + x + 2}$ at $x = 2$

$$f(2) = \frac{8}{4 + 2 + 2} = \frac{8}{8} = 1$$

$$f(x) = 8(x^2 + x + 2)^{-1}$$

$$f'(x) = -8(x^2 + x + 2)^{-2}(2x) = \frac{-16x}{(x^2 + x + 2)^2}$$

$$m = f'(2) = \frac{-32}{(4 + 2 + 2)^2} = \frac{-32}{8^2} = \frac{-32}{64} = -\frac{1}{2}$$

$$\boxed{y - 1 = -\frac{1}{2}(x - 2)}$$

Section 0.6

22) A college student earns income by typing term papers on a computer, which she leases (along with a printer). The student charges \$4 per page for her work, and she estimates that her monthly cost when typing x pages is $C(x) = .10x + 75$ dollars.

a) What is the student's profit if she types 100 pages in 1 month?

$$R(x) = 4x, \quad C(x) = .10x + 75 \quad *$$

$$P(x) = R(x) - C(x) = 4x - (.10x + 75) = 4x - 0.10x - 75 = 3.9x - 75$$

$$P(100) = 3.9(100) - 75 = 390 - 75 = \boxed{\$315}$$

24) A cellular telephone company estimates that, if it has x thousand subscribers, its monthly profit is $P(x)$ thousand dollars where $P(x) = 12x - 200$

a) How many subscribers are needed for a monthly profit of 160 thousand dollars?

$$160 = 12x - 200 \Rightarrow 360 = 12x \Rightarrow x = 30 \text{ thousand subscribers}$$

26) A catering company estimates that, if it has x customers in a typical week, its expenses will be approximately $c(x) = 550x + 6500$ dollars, and its revenue will be approximately $R(x) = 1200x$ dollars.

a) How much profit will the company earn in 1 week when it has 12 customers?

$$P(x) = R(x) - C(x) = 1200x - (550x + 6500) = 1200x - 550x - 6500 = 650x - 6500$$

$$P(12) = 650(12) - 6500 = 1300 \text{ dollars}$$

Section 1.7

38) Estimate the cost of manufacturing 51 bicycles per day in Exercise 37.

$$C(51) \approx C(50) + C'(51) = 5000 + 45 = \$5045$$

40) Let $P(x)$ be the profit from producing (and selling) x units of goods. Match each question with the proper solution.

Questions

A. What is the profit from producing 1000 units of goods?

B. At what level of production will the marginal profit be 1000 dollars?

C. What is the marginal profit from producing 1000 units of goods?

D. For what level of production will the profit be 1000 dollars? *

Solutions

(a) Compute $P'(1000)$

(b) Find a value of a for which $P'(a) = 1000$

(c) Set $P(x) = 1000$ and solve for x

(d) Compute $P(1000)$

A (d)

B (b)

C (a)

D (c)

classwork problems graded: cw4 #2c, #3 and cw5 #2