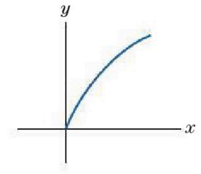
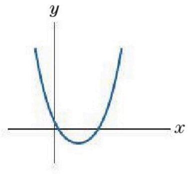
Homework 4

section 2.1

2) which functions are decreasing for all x

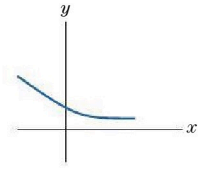


(a)

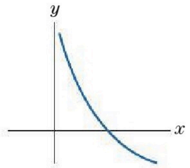


(b)

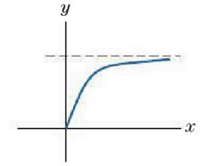
c, d



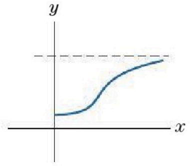
(c)



(d)

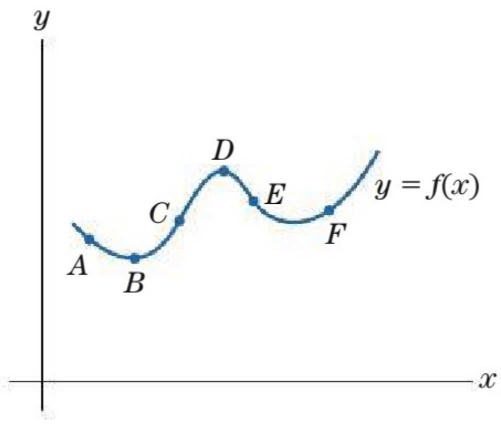


(e)



(f)

18)

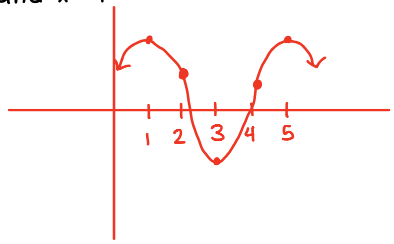


a) At which labeled points is the function decreasing?
b) At which labeled points is the function concave down?

a) A, E
b) D (also acceptable: C and E)

*

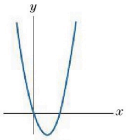
34) Sketch the graph of a function having the given properties:
Relative maximum points at x=1 and x=5; relative minimum point at x=3; inflection points at x=2 and x=4



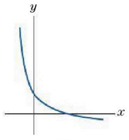
*

section 2.2

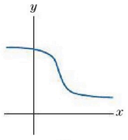
2) which functions have a negative first derivative for all x?



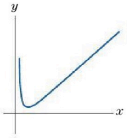
(a)



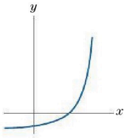
(b)



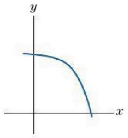
(c)



(d)



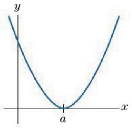
(e)



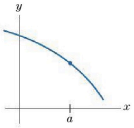
(f)

(b), (c), (f)

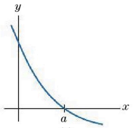
6) which one of the graphs could represent a function f(x) for which $f(a)=0$, $f'(a)<0$, and $f''(a)>0$



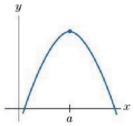
(a)



(b)



(c)



(d)

$f'(a)<0 \Rightarrow$ decreasing
$f''(a)>0 \Rightarrow$ concave up
point (a,0)

so $\boxed{\text{(c)}}$

section 2.3

26) Sketch the following curve. Indicate all relative extreme points and inflection points: $y = x^3 - 3x + 2$

$y = x^3 - 6x^2 + 9x + 3 \ = f(x)$

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1)$ *

Critical numbers: $x = 3, 1$

f': $+\ \underset{1}{|}\ -\ \underset{3}{|}\ +$ increasing on $(-\infty, 1), (3, \infty)$

decreasing on $(1, 3)$

relative max at $x = 1$: $f(1) = 1 - 6 + 9 + 3 = 7$ so point $(1, 7)$

relative min at $x = 3$: $f(3) = 27 - 6(9) + 27 + 3 = 54 - 54 + 3 = 3$ so point $(3, 3)$

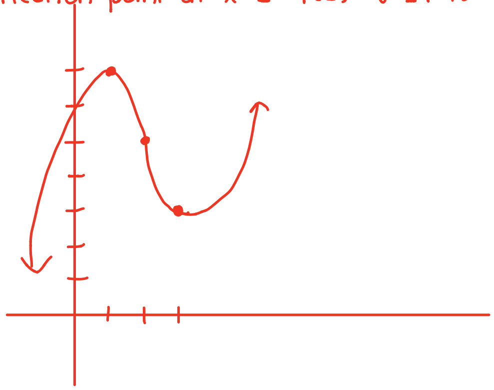
$f''(x) = 6x - 12 = 6(x-2)$

Important number: $x = 2$

f'': $-\ \underset{2}{|}\ +$ concave up: $(2, \infty)$

concave down: $(-\infty, 2)$

Inflection point at $x = 2$: $f(2) = 8 - 24 + 18 + 3 = 5$ so point $(2, 5)$



Section 2.4

12) Sketch the graph of $f(x)=x^3+2x^2+4x$

(1) domain: $(-\infty,\infty)$

(2) x-int: $0=x^3+2x^2+4x \Rightarrow 0=x(x^2+2x+4) \Rightarrow x=0$

$x^2+2x+4=0 \Rightarrow x=\frac{-2\pm\sqrt{4-4(1)(4)}}{2}=\frac{-2\pm\sqrt{4-16}}{2}$ ← square root of a negative so no solution

y-int: $f(0)=0$

(3)
$$\left.\begin{array}{l}\lim\limits_{x\to\infty}(x^3+2x^2+4x)=\lim\limits_{x\to\infty}x^3=\infty\\[2ex]\lim\limits_{x\to-\infty}(x^3+2x^2+4x)=\lim\limits_{x\to-\infty}x^3=-\infty\end{array}\right\}\text{no H.A.}$$

f is never undefined ⇒ no V.A.

(4) $f'(x)=3x^2+4x+4$

$0=3x^2+4x+4 \Rightarrow x=\frac{-4\pm\sqrt{16-4(3)(4)}}{6}$ ← square root of a negative so no critical points

$f' \ \overset{+}{\overline{\qquad\qquad\qquad}}$ so f is increasing on $(-\infty,\infty)$; no relative extrema

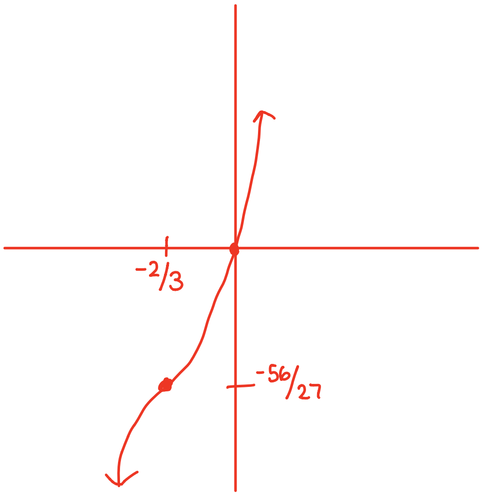
(5) $f''(x)=6x+4$

$0=6x+4 \Rightarrow 6x=-4 \Rightarrow x=-4/6=-2/3$

$f'': \ \overset{-\qquad+}{\underset{-2/3}{\overline{\qquad|\qquad}}}$ concave down on $(-\infty,-2/3)$
concave up on $(-2/3,\infty)$

Inflection point at $x=-2/3$

$f(-\frac{2}{3})=\frac{-8}{27}+2\left(\frac{4}{9}\right)+4\left(\frac{-2}{3}\right)=\frac{-8}{27}+\frac{8}{9}-\frac{8}{3}=\frac{-8}{27}+\frac{24}{27}-\frac{72}{27}=\frac{-56}{27}$



20) Sketch the graph of $f(x) = 3x^4 - 6x^2 + 3$

(1) domain: $(-\infty, \infty)$

(2) x-int: $0 = 3x^4 - 6x^2 + 3 = 3(x^4 - 2x^2 + 1) = 3(x^2-1)(x^2-1) = 3(x-1)(x+1)(x-1)(x+1) = 3(x-1)^2(x+1)^2$

$\Rightarrow x = \pm 1$

y-int: $f(0) = 3$

(3)
$$\left.\begin{array}{l} \lim\limits_{x\to\infty}(3x^4-6x^2+3) = \lim\limits_{x\to\infty} 3x^4 = \infty \\ \\ \lim\limits_{x\to-\infty}(3x^4-6x^2+3) = \lim\limits_{x\to-\infty} 3x^4 = \infty \end{array}\right\} \text{no H.A.}$$

f is never undefined so no V.A.

(4) $f'(x) = 12x^3 - 12x = 12x(x^2-1) = 12x(x-1)(x+1)$

critical numbers: $0, \pm 1$

$$f' \;\; \begin{array}{ccccccc} - & | & + & | & - & | & + \\ & -1 & & 0 & & 1 & \end{array}$$

increasing on: $(-1,0), (1,\infty)$

decreasing on: $(-\infty,-1), (0,1)$

relative min at $x=-1$, $x=1$: $f(-1) = 3-6+3 = 0$ $f(1) = 3-6+3 = 0$

relative max at $x=0$: $f(0)=3$

(5) $f''(x) = 36x^2 - 12 = 12(3x^2-1)$

$0 = 3x^2 - 1 \Rightarrow x^2 = 1/3 \Rightarrow x = \pm\sqrt{1/3}$

$$f'' \;\; \begin{array}{ccccc} + & | & - & | & + \\ & -\sqrt{1/3} & & \sqrt{1/3} & \end{array}$$

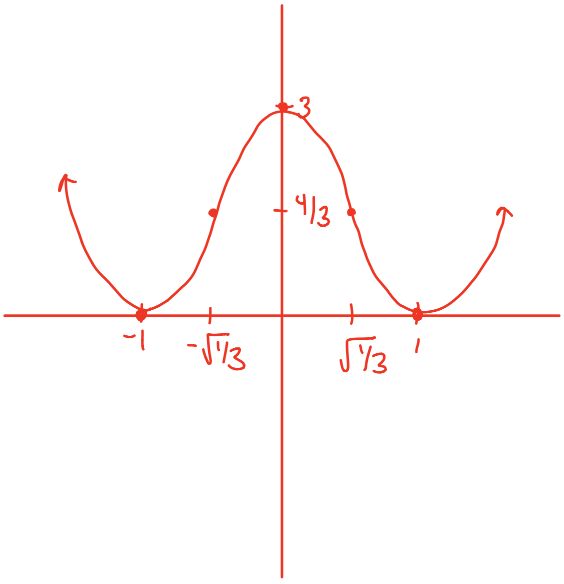
concave up: $(-\infty, -\sqrt{1/3}), (\sqrt{1/3}, \infty)$

concave down: $(-\sqrt{1/3}, \sqrt{1/3})$

inflection points at $x = \pm\sqrt{1/3}$

$$f\left(-\sqrt{\tfrac{1}{3}}\right) = 3\left(\tfrac{1}{3}\right)^2 - 6\left(\tfrac{1}{3}\right) + 3 = 3\left(\tfrac{1}{9}\right) - 2 + 3 = \tfrac{1}{3} + 1 = \tfrac{4}{3}$$

$$f\left(\sqrt{\tfrac{1}{3}}\right) = 3\left(\tfrac{1}{3}\right)^2 - 6\left(\tfrac{1}{3}\right) + 3 = 3\left(\tfrac{1}{9}\right) - 2 + 3 = \tfrac{1}{3} + 1 = \tfrac{4}{3}$$



classwork problems graded: CW6 #1, CW7 #1 and #2