Homework 5

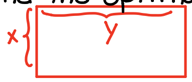
section 2.5

14) Consider the problem of finding the dimensions of the rectangular garden of area 100 square meters for which the amount of fencing needed to surround the garden is as small as possible.

a) Draw a picture of a rectangle and select appropriate letters for the dimensions.

b) Determine the objective and constraint equations

c) Find the optimal values for the dimensions

a)


b) constraint: $100 = xy$

objective: $\text{fencing} = f = 2x + 2y$

c) $100 = xy \Rightarrow y = \frac{100}{x} \Rightarrow f(x) = 2x + 2\left(\frac{100}{x}\right) = 2x + \frac{200}{x}$

$$f'(x) = 2 - \frac{200}{x^2}$$

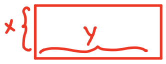
$$0 = 2 - \frac{200}{x^2} \Rightarrow 2 = \frac{200}{x^2} \Rightarrow 2x^2 = 200 \Rightarrow x^2 = 100 \Rightarrow x = 10$$

check this is a min:


✓

min at $x = 10m \Rightarrow y = \frac{100}{10} = 10m$

20) Find the dimensions of the rectangular garden of greatest area that can be fenced off (all four sides) with 300 meters of fencing.



constraint: $2x + 2y = 300$

objective: $A = xy$

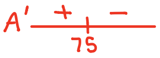
$2x + 2y = 300 \Rightarrow 2y = 300 - 2x \Rightarrow y = \frac{300-2x}{2} = 150 - x$

*

$\Rightarrow A = 150x - x^2$

$A'(x) = 150 - 2x$

$0 = 150 - 2x \Rightarrow 150 = 2x \Rightarrow x = 75$

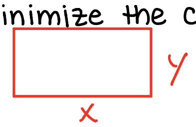
check this is a max:


✓

max at $x = 75m \Rightarrow y = 150 - 75 = 75m$

section 2.6

18) A supermarket is to be designed as a rectangular building with floor area 12,000 square feet. The front of the building will be mostly glass and will cost \$70 per running foot for materials. The other three walls will be constructed of brick and cement block, at a cost of \$50 per running foot. Ignore all other costs (labor, cost of foundation and roof, and the like) and find the dimensions of the base of the building that will minimize the cost of the materials for the four walls of the building.



constraint: $xy = 12000$

objective: $C = 70x + 50(2y+x)$

$$xy = 12000 \Rightarrow y = \frac{12000}{x} \Rightarrow C = 70x + 100\left(\frac{12000}{x}\right) + 50x = 120x + \frac{1200000}{x}$$

$$C'(x) = 120 - \frac{1200000}{x^2}$$

$$0 = 120 - \frac{1200000}{x^2} \Rightarrow \frac{1200000}{x^2} = 120 \Rightarrow 1200000 = 120x^2 \Rightarrow x^2 = 10000 \Rightarrow x = 100$$

Check this is a min: $\underset{100}{- \mid +}$

$$y = \frac{12000}{100} = 120$$

$\boxed{100\text{ft} \times 120\text{ft}}$

Section 2.7

12) The average ticket price for a concert at the opera house was \$50. The average attendance was 4000. When the ticket price was raised to \$52, attendance declined to an average 3800 persons per performance. What should the ticket price be to maximize the revenue for the opera house? (Assume a linear demand curve.)

two points: (4000, 50) and (3800, 52)

$$m = \frac{52-50}{3800-4000} = \frac{-2}{200} = \frac{-1}{100}$$

$$p - 50 = \frac{-1}{100}(x - 4000) \Rightarrow p - 50 = \frac{-1}{100}x + 40 \Rightarrow p = \frac{-1}{100}x + 90$$ *

$$R(x) = \frac{-1}{100}x^2 + 90x$$

$$R'(x) = \frac{-2}{100}x + 90 = \frac{-x}{50} + 90$$

$$0 = \frac{-x}{50} + 90 \Rightarrow \frac{x}{50} = 90 \Rightarrow x = 4500$$

$$p(4500) = \frac{-1}{100}(4500) + 90 = -45 + 90 = \boxed{\$45}$$

Classwork problem graded: 4 (out of 4 pts)