Homework 9

section 3.3

6) Suppose that x and y are related by the given equation and use implicit differentiation to determine dy/dx: $x^3+y^3=x^2+y^2$

$x^3+y^3=x^2+y^2 \Rightarrow 3x^2+3y^2\frac{dy}{dx}=2x+2y\frac{dy}{dx} \Rightarrow 3y^2\frac{dy}{dx}-2y\frac{dy}{dx}=2x-3x^2$

$\Rightarrow \frac{dy}{dx}(3y^2-2y)=2x-3x^2 \Rightarrow \frac{dy}{dx}=\boxed{\frac{2x-3x^2}{3y^2-2y}}$

18) Suppose that x and y are related by the given equation and use implicit differentiation to determine dy/dx: $x^3y+xy^3=4$

$3x^2y+x^3\frac{dy}{dx}+y^3+x(3y^2\frac{dy}{dx})=0$

$x^3\frac{dy}{dx}+3xy^2\frac{dy}{dx}=-3x^2y-y^3$

$\frac{dy}{dx}(x^3+3xy^2)=-3x^2y-y^3$

$\frac{dy}{dx}=\frac{-3x^2y-y^3}{x^3+3xy^2}$

40) Suppose that the price p (in dollars) and the weekly demand, x (in thousands of units) of a commodity satisfy the demand equation

$6p+x+xp=94$

How fast is the demand changing at a time when $x=4$, $p=9$, and the price is rising at the rate of \$2 per week?

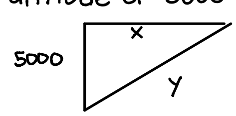
Given: $\frac{dp}{dt}=2$, $x=4$, $p=9$

$6p+x+xp=94 \Rightarrow 6\frac{dp}{dt}+\frac{dx}{dt}+x\frac{dp}{dt}+\frac{dx}{dt}p=0$

$\Rightarrow 6(2)+\frac{dx}{dt}+4(2)+9\frac{dx}{dt}=0 \Rightarrow 10\frac{dx}{dt}=-20 \Rightarrow \frac{dx}{dt}=$ $\boxed{-2 \text{ thousand units/week}}$

44) An airplane flying 390 feet per second at an altitude of 5000 feet flew directly over an observer.

a) Find an equation relating x and y



b) Find the value of x when y is 13,000

c) How fast is the distance from the observer to the airplane changing at the time when the airplane is 13,000 feet from the observer? That is, what is dy/dt at the time when $dx/dt=390$ and $y=13{,}000$

a) $x^2+5000^2=y^2$

b) $x^2+5000^2=(13000)^2 \Rightarrow x^2=144000000 \Rightarrow x=12000$

c) $2x\frac{dx}{dt}+0=2y\frac{dy}{dt} \Rightarrow 2(12000)(390)=2(13000)\frac{dy}{dt} \Rightarrow 9360000=26000\frac{dy}{dt}$

$\Rightarrow \frac{dy}{dt}=360$ ft/sec

section 6.1

13) Determine $\int \left(\frac{2}{x}+\frac{x}{2}\right) dx$

$$\int\left(\frac{2}{x}+\frac{x}{2}\right)dx = \int(2x^{-1}+\frac{1}{2}x)\,dx = 2\ln|x|+\frac{1}{2}\,\frac{x^2}{2}+C = \boxed{2\ln|x|+\frac{x^2}{4}+C}$$

16) Determine $\int\left(\frac{2}{\sqrt{x}}+2\sqrt{x}\right)dx$

$$\int\left(\frac{2}{\sqrt{x}}+2\sqrt{x}\right)dx = \int(2x^{-1/2}+2x^{1/2})dx = 2\left(\frac{x^{1/2}}{1/2}\right)+2\left(\frac{x^{3/2}}{3/2}\right)+C = 4x^{1/2}+\frac{4}{3}x^{3/2}+C$$

20) Determine $\int e^{-x}dx$

$$\int e^{-x}dx = \frac{e^{-x}}{-1}+C = \boxed{-e^{-x}+C}$$

section 9.1

8) Determine $\int \frac{(1+\ln x)^3}{x}dx$

$u=1+\ln x \Rightarrow du=\frac{1}{x}dx \Rightarrow dx = x\,du$

$$\int\frac{(1+\ln x)^3}{x}dx = \int\frac{u^3}{\cancel{x}}\cancel{x}\,du = \int u^3 du = \frac{u^4}{4}+C = \boxed{\frac{(1+\ln x)^4}{4}+C}$$

16) Determine $\int\frac{x}{\sqrt{x^2+1}}dx$

$u=x^2+1 \Rightarrow du=2x\,dx \Rightarrow dx=\frac{du}{2x}$

$$\int\frac{\cancel{x}}{\sqrt{u}}\,\frac{du}{2\cancel{x}} = \int\frac{1}{2}u^{-1/2}du = \frac{\cancel{1}}{\cancel{2}}\,\frac{u^{1/2}}{\cancel{1/2}}+C = \boxed{\sqrt{x^2+1}+C}$$

30) Determine $\int e^x\sqrt{1+e^x}\,dx$

$u=1+e^x \Rightarrow du=e^x dx \Rightarrow dx=\frac{du}{e^x}$

$$\int e^x\sqrt{1+e^x}\,dx = \int\cancel{e^x}\sqrt{u}\,\frac{du}{\cancel{e^x}} = \int u^{1/2}du = \frac{u^{3/2}}{3/2}+C = \frac{2}{3}u^{3/2}+C = \boxed{\frac{2}{3}(1+e^x)^{3/2}+C}$$