Math 203 Midterm 1 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, and homework problems.

(1) Use the limit definition of a derivative to find the derivative of each of the following functions:

(a) $g(t) = 5t^2$

(b) $h(s) = s^2 - 3s$

(2) Let $f(x) = \frac{1}{\sqrt[5]{x}}$. Find the equation for the tangent line to $f(x)$ at the point $(-1, -1)$.

(3) Consider the function $f(x) = 5 - x^2$.

(a) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1, 4)$.

(b) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2, 1)$.

(4) Find the indicated limit.

(a) $\lim\limits_{x \to 3} 4x^3$

(b) $\lim\limits_{x \to 0} \dfrac{1}{x^3 - 1} + 1$

(c) $\lim\limits_{x \to 1} \dfrac{3x - 4}{x^2 + x + 1}$

(d) $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x^3 + 1}$

(e) $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$

(f) $\lim\limits_{x \to \infty} \dfrac{2x^3 - 4x^2 + 5x}{17x^2 + 1}$

(g) $\lim\limits_{x \to -\infty} \dfrac{3x^2 + 4}{x + 7}$

(5) Find the derivative. Show your solution step-by-step. You don't need to simplify your answer.

(a) $f(x) = 7x^4 - \frac{4}{3}x^3 + \frac{1}{10}x^2 - \frac{8}{9}x + 40$

(b) $f(x) = \sqrt[4]{x^3}$

(c) $g(x) = \frac{6}{x^5} - \frac{8}{x}$

(6) On which interval(s) is the following function continuous. Justify your answers using limits:

$$f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \leq x \leq 1 \\ -3 & x > 1 \end{cases}$$

(7) For each function:

(i) Find all critical numbers.

(ii) Determine whether each critical number is a relative maximum, relative minimum, or neither.

(iii) Find all inflection points for $f(x)$.

(iv) Sketch a graph of $f(x)$. Label the (x, y)-coordinates of each extremum and the inflection point(s) for full credit.

(a) $f(x) = x^2 - 6x + 9$

(b) $f(x) = 2x^2 - 7x + 3$

(c) $f(x) = x^3 - x^2$

(d) $f(x) = 3x^4 - 12x^3$

(8) Consider the function $f(x)$ given below. Find

(i) $\lim_{x \to k^-} f(x)$

(ii) $\lim_{x \to k^+} f(x)$

(iii) $\lim_{x \to k} f(x)$

(iv) $f(k)$

(v) Is $f(x)$ continuous at k? (yes or no)

for each of the given values of k. If the given value does not exist, write "DNE", ∞, $-\infty$, or "undefined" as necessary:

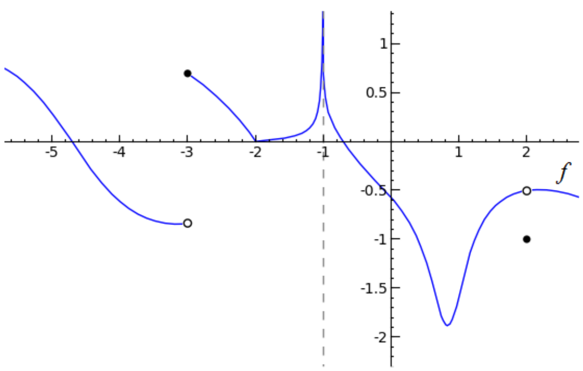

Figure 1: There is an asymptote at $x = -1$

(a) $k = -3$

(b) $k = -2$

(c) $k = -1$

(d) $k = 2$

Word Problems.

(9) A tiny particle is moving along the x-axis. The position of the particle at time t is given by the function

$$s(t) = -\frac{1}{3}t^3 + t$$

(a) Find a function $v(t)$ for the velocity of the particle at time t. Use your answer to find the velocity of the particle when $t = 0$.

(b) Find a function $a(t)$ for the acceleration of the particle at time t. Use your answer to find the rate at which the particle is accelerating when $t = 0$.

(10) A company does extensive market research to determine the optimal price for their product. They find out that:

- If the price is $d = \$50$, the demand is $x = 600$ units in a week.
- If the price is $d = \$40$, the demand is $x = 800$ units in a week.

Find a linear function $d(x)$ that expresses the price d in terms of the demand x (i.e. the demand equation). Use this to find the revenue function.

(11) A company's total sales (in millions of dollars) t months from now are $S(t) = 2\sqrt{t} + 5$.

(a) Find $S'(t)$.

(b) Find $S(25)$ and $S'(25)$. Interpret the meaning of these two numbers.

(12) A fence is to be built around a rectangular garden. The garden requires 100 square yards of area. Find the dimensions of the garden (length and width) that minimize the amount of fencing used.

(13) A fence is to be built around a (different) rectangular garden. One side of the fence is to be built with brick, costing \$35.00 per yard. The other three sides are to be built using wooden planks, costing \$25.00 per yard. The yard still requires 100 square yards of area. Find the dimensions that minimizes the cost of constructing the fence.

(14) The price function for a particular commodity with demand level x is $p(x) = 9 - 0.03x$. Find the production level that maximizes the revenue.