Math 203 Midterm 2 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, and homework problems.

(1) Find two numbers whose sum is 32 and whose product is maximized.

(2) Find two positive numbers whose product is 100, and whose sum is minimized.

(3) A company that produces electronic components expects to sell 4000 units this year. They can produce the units in production runs, but each production run costs \$250 to set up. After each production, it has to pay \$2.00 per unit to store the inventory, until they sell out (inventory cost is based on the *average* inventory between production runs). How many production runs should the company do in order to minimize the Inventory costs?

(4) Air is being pumped into a spherical balloon at a rate of 7 cm^3 per second. How fast is the *radius* of the balloon increasing at the instant the *volume* equals 36π? (The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$; you may write your answer in terms of π if you like).

(5) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(6) An ice cube is melting at a rate of 4cm^3 per minute. How fast is the length of the side decreasing when the side is 2cm? Recall the volume of a cube with side x is $V = x^3$.

(7) Suppose the price p and demand x of a commodity are related by the equation $100x^2 + 9p^2 = 3600$. Due to a surplus, the price p of the commodity is dropping at a rate of \$0.15 per week, causing the demand to increase. How fast is the demand increasing when the price p is equal to \$15?

(8) Calculate the derivative of each function. Show your work.

(a) $f(x) = 3e^{2x}$

(b) $g(x) = e^{-x^2}$

(c) $G(x) = 2xe^{2x}$

(d) $h(t) = \dfrac{t}{\ln t}$

(e) $j(u) = 15(1 - e^u)$

(f) $J(s) = 15(1 - e^{s^2-2s+1})$

(g) $h(t) = \dfrac{\ln t}{t}$

(h) $R(x) = \ln(x^2 + 10)$

(i) $j(x) = \ln(\ln x)$

(j) $g(x) = e^{x^2}e^x$

(9) Use logarithmic differentiation to find the derivative of

(a) $y = \dfrac{x^{2/3}(x-3)^{4/3}}{(2x+5)^{5/3}}$

(b) $y = \dfrac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$

(c) $y = \dfrac{(x^3-1)(x+2)^{1/2}}{x+1}$

(10) Evaluate the given integral.

(a) $\int 3x^2 + 3x + 9 + \dfrac{1}{x^3}\, dx.$

(b) $\int_1^4 \dfrac{1}{\sqrt{x}}\, dx$

(c) $\int_0^2 e^{-x}\, dx$

(d) $\int_{-2}^{2} e^{5x} + 5x^2\, dx$

(e) $\int \dfrac{x}{\sqrt{4x^2+9}}\, dx$

(f) $\int_{-1}^{1} x\sqrt{x^2+1}\, dx$

(g) $\int \left(\dfrac{\sqrt{x}}{2} + \dfrac{2}{\sqrt{x}}\right) dx$

(h) $\int x^{-3}(x+1)\, dx$

(i) $\int_{-1}^{1} t^3(1+t^4)^3\, dt$

(11) Find the area of the region shaded below. (the function is $f(x) = -(x-1)^2 + 1$).