Midterm 2 – Math 203

Tuesday, April 2, 2019

This is a closed-book exam.
You are allowed one cheat sheet (8.5 inch by 11 inch 2-sided)
Non-graphing calculators allowed.
Justify your answers to obtain full credit (and partial credit, too).
You have 75 minutes.
This exam consists of 7 questions.
Please verify that you have all pages.

Name:________________________________ **ID#:**________________

(this page intentionally left blank)

1. (*15 points*) You sell a total of 20 crates of ice cream each month. Storing the ice cream \$8 per crate (based on the average number of crates). To make a new order of crates costs \$5 per order. Let x represent the number of ice cream crates in each order and r represent the number of orders.

(a) Find a function for the total inventory cost (carrying cost + ordering costs) associated with ordering and storing the crates of ice cream.

(b) Find the amount of crates per order you should make to minimize the inventory cost. How many orders per month is that?

Solution

(a)

$$C = 5r + 8\left(\frac{x}{2}\right) = \boxed{5r + 4x}$$

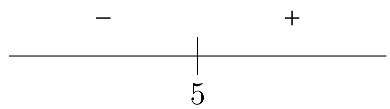
(b) We know that $xr = 20 \Rightarrow r = \dfrac{20}{x}$. Plugging this into the equation above we get

$$\begin{aligned} C(x) &= 5\left(\frac{20}{x}\right) + 4x \\ &= \frac{100}{x} + 4x \\ &= 100x^{-1} + 4x \end{aligned}$$

Taking the derivative and solving for the critical point we have:

$$\begin{aligned} C'(x) = 0 &\Leftrightarrow -100x^{-2} + 4 = 0 \\ &\Leftrightarrow \frac{100}{x^2} = 4 \\ &\Leftrightarrow 4x^2 = 100 \\ &\Leftrightarrow x^2 = 25 \\ &\Rightarrow x = 5 \end{aligned}$$

Checking that this is indeed a minimum we have



Thus the amount of crates per order should be $\boxed{\text{5 crates}}$. This would be $^{20}/_{5}$ = $\boxed{\text{4 orders}}$

$\square$

2. (*10 points*) A spherical snowball is placed in the sun. The sun melts the snowball so that its volume **decreases** 16π inches per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 inches.

The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Solution This problem had typos. The correct problem should read: A spherical snowball is placed in the sun. The sun melts the snowball so that its volume **decreases** 16π cubic inches per hour. Find the rate of change of the radius with respect to time at the instant the radius is 4 inches.

Everyone got 10 points for this problem.

□

3. (*20 points*) Evaluate the following integrals. **(Don't forget +C when necessary!)**

(a) $\int \left(e^{17x} + x^{3/4}\right)\, dx$

(b) $\int_1^e \left(9x^2 + x^{-1}\right)\, dx$ (Recall: $\ln(e) = 1$ and $\ln 1 = 0$.)

(c) $\int_1^{16} \frac{1}{\sqrt{x}}\, dx$

(d) $\int \left(7x + 4x^3 + \frac{1}{x^3}\right)\, dx$

Solution

(a)

$$\int \left(e^{17x} + x^{3/4}\right)\, dx = \frac{e^{17x}}{17} + \frac{x^{7/4}}{7/4} + C$$

$$= \boxed{\frac{e^{17x}}{17} + \frac{4}{7}e^{7/4} + C}$$

(b)

$$\int_1^e \left(9x^2 + x^{-1}\right)\, dx = \left[9\left(\frac{x^3}{3}\right) + \ln|x|\right]\Big|_1^e$$

$$= \left(3x^3 + \ln|x|\right)\Big|_1^e$$

$$= (3e^3 + \ln e) - (3(1)^3 + \ln 1)$$

$$= 3e^3 + \ln e - 3 - \ln 1$$

$$= 3e^3 + 1 - 3$$

$$= \boxed{3e^3 - 2}$$

(c)

$$\int_1^{16} \frac{1}{\sqrt{x}}\ dx = \int_1^{16} x^{-1/2}\ dx$$
$$= \left.\frac{x^{1/2}}{1/2}\right|_1^{16}$$
$$= 2\sqrt{x}\Big|_1^{16}$$
$$= 2\sqrt{16} - 2\sqrt{1}$$
$$= 8 - 2$$
$$= \boxed{6}$$

(d)

$$\int \left(7x + 4x^3 + \frac{1}{x^3}\right)\ dx = \int (7x + 4x^3 + x^{-3})]dx$$
$$= 7\left(\frac{x^2}{2}\right) + 4\left(\frac{x^4}{4}\right) + \frac{x^{-2}}{-2} + C$$
$$= \boxed{\frac{7x^2}{2} + x^4 - \frac{1}{2x^2} + C}$$

□

4. (*20 points*) Find $\frac{dy}{dx}$ for the following functions. **You do not have to simplify your answer.**

(a) $y = \ln(4x^2 + 5x)$

(b) $y = 2x^3 e^x$

(c) $y = e^{4x^4+7x^2+4}$

(d) $y = \dfrac{\ln x}{4e^x + x^2}$

Solution

(a)

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{4x^2+5x} \cdot \frac{d}{dx}(4x^2+5x) \\ &= \boxed{\frac{1}{4x^2+5x}(8x+5)}\end{aligned}$$

(b)

$$\begin{aligned}\frac{dy}{dx} &= \left(\frac{d}{dx}(2x^3)\right)e^x + 2x^3\left(\frac{d}{dx}(e^x)\right) \\ &= \boxed{6x^2e^x + 2x^3e^x}\end{aligned}$$

(c)

$$\begin{aligned}\frac{dy}{dx} &= e^{4x^4+7x^2+4} \cdot \frac{d}{dx}(4x^4+7x^2+4) \\ &= \boxed{e^{4x^4+7x^2+4}(16x^3+14x)}\end{aligned}$$

(d)

$$\frac{dy}{dx} = \frac{(4e^x + x^2)(\frac{d}{dx}(\ln x) - (\ln x)(\frac{d}{dx}(4e^x + x^2))}{(4e^x + x^2)^2}$$
$$= \boxed{\frac{(4e^x + x^2)(\frac{1}{x}) - (\ln x)(4e^x + 2x)}{(4e^x + x^2)^2}}$$

□

5. (*10 points*) Use logarithmic differentiation to find the derivative of $y = \dfrac{(x+1)^5(2x^3+5)^3}{\sqrt{2x+4}}$. You do not need to simplify your answer.

Solution

$$\begin{aligned}\ln y &= \ln \frac{(x+1)^5(2x^3+5)^3}{(2x+4)^{1/2}} \\ &= \ln(x+1)^5 + \ln(2x^3+5)^3 - \ln(2x+4)^{1/2} \\ &= 5\ln(x+1) + 3\ln(2x^3+5) - \frac{1}{2}\ln(2x+4)\end{aligned}$$

Differentiating this we get

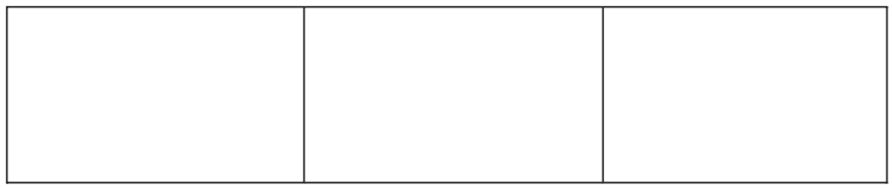
$$\begin{aligned}&5\left(\frac{1}{x+1}\right) + 3\left(\frac{1}{2x^3+5}\cdot\frac{d}{dx}(2x^3+5)\right) - \frac{1}{2}\left(\frac{1}{2x+4}\cdot\frac{d}{dx}(2x+4)\right) \\ &= \frac{5}{x+1} + \frac{3}{2x^3+5}(6x^2) - \frac{1}{2(2x+4)}(2) \\ &= \frac{5}{x+1} + \frac{18x^2}{2x^3+5} - \frac{2}{2(2x+4)}\end{aligned}$$

Thus

$$\frac{dy}{dx} = \boxed{\left(\frac{(x+1)^5(2x^3+5)^3}{\sqrt{2x+4}}\right)\left(\frac{5}{x+1} + \frac{18x^2}{2x^3+5} - \frac{2}{2(2x+4)}\right)}$$

□

6. (*15 points*) An ecologist is conducting a research project on breeding pheasants in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the sketch. Find the **dimensions** of the pen that has the maximum area she can enclose with 2400 m of fencing.



Solution Letting the width be x and the lengths be y we have

$$2400 = 4x + 2y \Rightarrow 2y = 2400 - 4x \Rightarrow y = 1200 - 2x$$

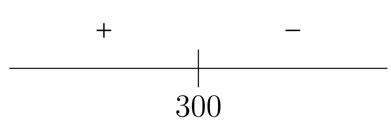
Plugging this into the equation for area we have

$$A = xy = x(1200 - 2x) = 1200x - 2x^2$$

Taking the derivative and solving for the critical point we have

$$\begin{aligned} A'(x) = 0 &\Leftrightarrow 1200 - 4x = 0 \\ &\Leftrightarrow 4x = 1200 \\ &\Leftrightarrow x = 300 \end{aligned}$$

Checking this is a max we have



At $x = 300$, $y = 1200 - 2(300) = 600$. Thus the dimensions are $\boxed{300 \text{ m x } 600 \text{ m}}$

□

7. (*10 points*) Find an equation of the line tangent to $x^2y + y^4 = 4 + 2x$ at $(-1, 1)$

Solution

$$x^2y + y^4 = 4 + 2x \Rightarrow 2xy + x^2\frac{dy}{dx} + 4y^3\frac{dy}{dx} = 2$$

Plugging in the point we have

$$\begin{aligned} 2(-1)(1) + (-1)^2\frac{dy}{dx} + 4(1)^3\frac{dy}{dx} = 2 &\Leftrightarrow -2 + \frac{dy}{dx} + 4\frac{dy}{dx} = 2 \\ &\Leftrightarrow 5\frac{dy}{dx} = 4 \\ &\Leftrightarrow \frac{dy}{dx} = \frac{4}{5} \end{aligned}$$

Using point slope form we have $\boxed{y - 1 = \frac{4}{5}(x + 1)}$

$\square$

Final Score

	Score	Out of
Question 1		15
Question 2		10
Question 3		20
Question 4		20
Question 5		10
Question 6		15
Question 7		10
Total		100