Math 241 Spring 2019 Professor MG
Classwork 7 Name: _____

Show all work and circle/box your final answer. All answers must be simplified unless stated otherwise.

1. Consider the function $f(x) = 3x^4 - 4x^3$
   (a) Find the open intervals where f is increasing and the intervals where f is decreasing.
   (b) Find both coordinates of any local extrema of the graph of f.
   (c) Find the intervals where f is concave up, and the intervals where f is concave down.
   (d) Find both coordinates of the inflection points of f.
   (e) Using the above information, sketch the graph of $y = f(x)$ on the coordinate axes below. You must label both coordinates of any local extrema and inflection points on your graph. (The graph does not need to be to scale.)

2. Let
$$f(x) = \frac{1}{1+x^2}$$
   Then
$$f'(x) = -\frac{2x}{(1+x^2)^2} \text{ and } f''(x) = \frac{6x^2-2}{(1+x^2)^3}$$
   (a) Find the intervals on which the graph is increasing and decreasing. Find the local maximums and minimums.
   (b) Find the intervals on which the graph is concave up and which it is concave down. Find any inflection points.

3. Sketch the graph of a single function that has all of the properties listed:
   (a) Continuous for all real numbers
   (b) $f'(x) > 0$ on $(-\infty, -2)$ and $(0, 3)$
   (c) $f'(x) < 0$ on $(-2, 0)$ and $(3, \infty)$
   (d) $f''(x) < 0$ on $(-\infty, 0)$ and $(0, 5)$
   (e) $f''(x) > 0$ on $(5, \infty)$
   (f) $f'(-2) = f'(3) = 0$
   (g) $f'(0)$ doesn't exist
   (h) Differentiable everywhere except at $x = 0$
   (i) An inflection point at $(5, 1)$