

Math 241 Midterm 1 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

(1) Calculate the following limits.

(a) $\lim_{t \rightarrow 1} \frac{t^2 + t - 2}{t^2 - 1}$

(h) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

(b) $\lim_{x \rightarrow 2} \frac{\sqrt{x^2 + 12} - 4}{x - 2}$

(i) $\lim_{x \rightarrow -3} \sqrt{x - 5}$

(c) $\lim_{x \rightarrow 3} 4x^3$

(j) $\lim_{t \rightarrow 2^-} \frac{t + 2}{t - 2}$

(d) $\lim_{x \rightarrow 0} \frac{1}{x^3 - 1} + 1$

(k) $\lim_{x \rightarrow 0} \frac{\sin x}{3x}$

(e) $\lim_{x \rightarrow 1} \frac{3x - 4}{x^2 + x + 1}$

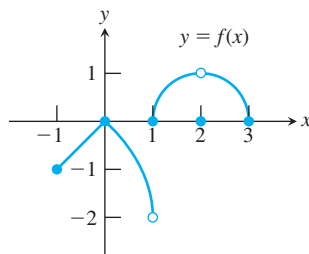
(l) $\lim_{x \rightarrow 0} \frac{x \cos\left(\frac{1}{x}\right)}{4}$

(f) $\lim_{x \rightarrow -1^-} \frac{|x + 1|}{x + 1}$

(g) $\lim_{x \rightarrow 2^+} \frac{8x - 16}{|x - 2|}$

(m) $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

(2) Find the following limits for



(a) $\lim_{x \rightarrow -1^+} f(x)$

(b) $\lim_{x \rightarrow 1^-} f(x)$

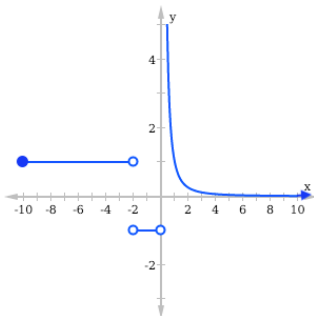
(c) $\lim_{x \rightarrow 2} f(x)$

(3) Describe on which intervals the following functions are continuous:

$$(a) f(x) = \begin{cases} 3 - x, & x < 2 \\ \frac{x}{2} + 1, & 2 \leq x < 4 \\ 3, & x \geq 4 \end{cases}$$

$$(b) f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \leq x \leq 1 \\ -3 & x > 1 \end{cases}$$

(c)



(4) Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$. Make sure to explain what theorem(s) you are using and how.

(5) Use the definition of the derivative (limit definition) to find the derivatives of the following:

(a) $f(x) = \sqrt{x}$

(b) $f(x) = x^2 - x$

(c) $f(x) = \frac{1}{x}$

(6) Consider the function $f(x) = 5 - x^2$.

(a) Find $f'(x)$ using the definition of a derivative.

(b) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1, 4)$.

(c) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2, 1)$.

(7) Find the first derivatives of the following:

(a) $y = 6x^2 - 10x - 5x^{-2}$

(b) $y = x^2 \sin x + 2x \cos x - 2 \sin x$

(c) $h(x) = x \tan(2\sqrt{x}) + 7$

(d) $y = \frac{\cot x}{1 + \cot(x^2 + x)}$

(e) $y = \left(1 - \frac{x}{7}\right)^{-7}$

(8) Find $\frac{dy}{dx}$ if $y \sin x = 1 - xy$.

(9) Find equations for the tangent and normal lines to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1, 0)$.