Math 241 Midterm 2 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures, classwork, quizzes, and homework problems.

(1) If a particle's motion is given by the equation $s(t) = 4t^3 - 10t^2 + 5$, find its velocity and acceleration as functions of t. What is its speed at $t = 1$

(2) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(3) A rectangular plot of land will be bounded on one side by a river and on the other three sides by some sort of fence. With 800 m of fencing at your disposal, what is the largest area you can enclose, and what are its dimensions?

(4) A child flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from them at a rate of 25 ft/sec. How fast must they let out the string when the kite is 500 ft away from them?

(5) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radios 10 cm. What is the maximum volume?

(6) For the following functions, **a)** find the critical points, **b)** classify them as local maxima, local minima, or neither, **c)** find where the function is increasing, **d)** find where the function is concave up, and **e)** sketch the graph.

   (a) $y = x^4 - 2x^2$

   (b) $y = x^5 - 5x^4$

(7) Given

$$f(x) = \frac{(x+1)(x+3)}{x^2+3} \qquad f'(x) = \frac{4(3-x^2)}{(x^2+3)^2} \qquad f''(x) = \frac{8x(x^2-9)}{(x^2+3)^3}$$

   (a) List all x and y intercepts

   (b) Find the intervals of increase and decrease

   (c) Find the intervals of concavity and any inflection points.

   (d) Find any asymptoptes

   (e) Sketch the graph

(8) Find the absolute maximum and minimum values of the following functions of the given intervals.

   (a) $f(x) = x^2 - 1, \ -1 \le x \le 2$

   (b) $f(x) = \sqrt[3]{x}, \ -1 \le x \le 8$

(9) Evaluate the following integrals

(a) $\int_1^4 \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}} \right) \, dx$

(b) $\int_1^2 x^{-3}(x+1) \, dx$

(c) $\int_0^{\pi/3} 2\sec^2 x \, dx$

(10) Solve the initial value problem

(a) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, $y(4) = 0$

(b) $\frac{ds}{dt} = 12t(3t^2 - 1)^2$, $s(1) = 3$

(11) The acceleration of an object is given by $\frac{3t}{8}$ find the position given that $v(4) = 3$ and $s(4) = 4$.

(12) Using 4 rectangles of equal length and the following rules find Riemann sums estimates for $f(x) = -x^2 + 16$ from $x = -2$ to $x = 2$ (i.e. to estimate $\int_{-2}^{2}(-x^2 + 16) \, dx$).

(a) Left-hand endpoints

(b) Right-hand endpoints

(c) Midpoints

(13) Find $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$

(a) by evaluating the integral and differentiating the result.

(b) by differentiating the integral directly

(14) Find $g'(x)$ if

$$g(x) = \int_x^{x^2} \frac{1}{t^3 + 1} \, dt$$

(15) Use linear approximation to estimate the following numbers (you do not need to simplify your answers):

(a) $(.95)^{10}$

(b) $\sqrt{10}$

(c) $\frac{1}{101}$ (using that $1/100 = 0.01$)

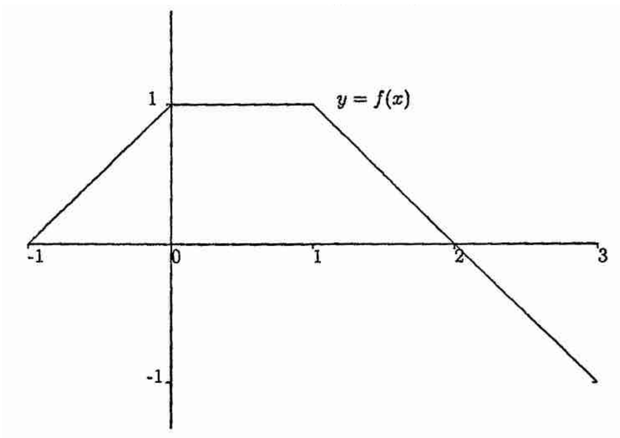
(d) $29^{1/3}$

(16) The graph of a function f is given below. Let

$$g(x) = \int_0^x f(t)\ dt$$

(a) Find $g(0)$

(b) Find $g(3)$



(17) Show that $1 + x = x^3$ has exactly one solution in the interval $[1, 2]$

(18) Show that $f(x) = x^4 + 3x + 1$ has exactly one zero in $[-1, -1]$

Answers:

(1) 8

(2) (a) 14 cm^2/sec, increasing
 (b) 0 cm/sec neither increasing nor decreasing
 (c) -14/13 cm/sec, decreasing

(3) 200 m by 400 m, 80000 m^2

(4) 20 ft/sec

(5) $h = 20/\sqrt{3}$, $r = 100 - ({}^{20}\!/\!{\sqrt{3}})^2/4$, $V = 4000\pi/3\sqrt{3}$

(6) (a) i. -1,0,1
 ii. rel min: $(\pm 1, -1)$, rel max: $(0,0)$
 iii. increasing on $(-1,0),(1,\infty)$ decreasing on $(-\infty,-1),(0,1)$
 iv. concave up on $(-\infty, -\sqrt{1/3}), (\sqrt{1/3}, \infty)$, infection points: $(-\sqrt{1/3}, -5/9)$, $(\sqrt{1/3}, -5/9)$
 v. Use an online graphing utility
 (b) i. 0,4
 ii. increasing on $(-\infty,0)$, $(4,\infty)$ decreasing on $(0,4)$
 iii. concave down on $(-\infty,0),(0,3)$ and concave up on $(3,\infty)$
 iv. Use an online graphing utility

(7) (a) $x = -1, -3$, $y = 1$
 (b) increasing on $(-\sqrt{3}, \sqrt{3})$, decreasing on $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
 (c) concave up on $(-3,0),(3,\infty)$, concave down on $(-\infty,-3)$, $(0,3)$, inflection points at $x = -3, 0, 3$
 (d) No vertical asymptotes, horizontal asymptote $y = 1$
 (e) Use a graphing utility

(8) (a) absolute max: 3, absolute min: -1
 (b) absolute max: 2, absolute min: -1

(9) (a) $19/3$
 (b) $7/8$
 (c) $2\sqrt{3}$

(10) (a) $y = \sqrt{x} - 2$
 (b) $y = 18x^6 - 18x^4 + 6x^2 - 3$

(11) $s(t) = t^3/16$

(12) (a) 58
 (b) 58
 (c) 59

(13) (a) $\cos\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)$

(b) $\cos\sqrt{x}\left(\frac{1}{2\sqrt{x}}\right)$

(14) $-\frac{1}{x^3+1}+\frac{2x}{(x^2)^3+1}$

(15) (a) 0.5

(b) $^{19}/_{6}$

(c) $^{99}/_{10000}$

(d) $^{83}/_{27}$

(16) (a) 0

(b) 1

(17) Use intermediate value theorem and mean value theorem

(18) Use intermediate value theorem and mean value theorem