The following are true statements involving chapters 2 and 3. Prove these statements by working through the problem and showing all necessary steps.

(1) $\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right)=-\frac{1}{2}$

(2) $g(x)=\begin{cases}\frac{4-x^2}{2+x} & x<1\\ 1 & x=1\\ 2-x^2 & 1<x\leq 2\\ x-3 & x>2\end{cases}$ is continuous at $x=1$ but not at $x=2$

(3) $\sqrt{x-5}=\frac{1}{x+3}$ has at least one solution.

(4) $\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}=-4$

(5) The equation of the line tangent to $y=3+4x^2-2x^3$ at $x=1$ is $y=2x+3$ and can be found using the limit definition of a derivative.

(6) $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=-\frac{3}{5}$ for $\frac{5x}{1+x^2}$

(7) $y=\frac{x^2+4x+3}{\sqrt{x}}\Rightarrow\frac{dy}{dx}=\frac{3x^2+4x-3}{2x^{3/2}}$

(8) $F(y)=\left(\frac{1}{y^2}-\frac{3}{y^4}\right)(y+5y^3)\Rightarrow F'(y)=\frac{9}{y^4}+\frac{14}{y^2}+5$

(9) $\frac{d}{dx}[f(x)g(x)h(x)]=f'(x)g(x)h(x)+f(x)g'(x)h(x)+f(x)g(x)h'(x)$

(10) $H(\theta)=\theta\sin\theta\Rightarrow H''(\theta)=2\cos\theta-\theta\sin\theta$

(11) $\lim_{t\to 0}\frac{\tan(6t)}{\sin(2t)}=3$

(12) $\tan\left(\frac{x}{y}\right)=x+y\Rightarrow\frac{dy}{dx}=\frac{y\sec^2\left(\frac{x}{y}\right)-y^2}{y^2+x\sec^2\left(\frac{x}{y}\right)}$

(13) If a particle has position function $s(t)=t^3-12t^2+36t$, then it is speeding up on the intervals $(2,4)$, $(6,\infty)$. and slowing down on the intervals $[0,2]$, $(4,6)$.

(14) If two cars start moving from the same point with Car A traveling south at 60mph and Car B traveling west at 25mph, then their distance is increasing at a rate of 65mph 2 hours later.