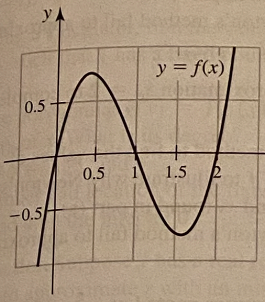
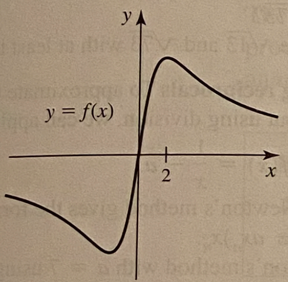
5. Let $f(x) = 2x^3 - 6x^2 + 4x$. Use Newton's method to find x_1 given that $x_0 = 1.4$. Use the graph of f (see figure) and an appropriate tangent line to illustrate how x_1 is obtained from x_0.



6. The function $f(x) = \dfrac{4x}{x^2 + 4}$ is differentiable and has a local maximum at $x = 2$, where $f'(2) = 0$ (see figure).
   a. Use a graphical explanation to show that Newton's method applied to $f(x) = 0$ fails to produce a value x_1 for the initial approximation $x_0 = 2$.
   b. Use the formula for Newton's method to show that Newton's method fails to produce a value x_1 with the initial approximation $x_0 = 2$.



7. How do you decide when to terminate Newton's method?

8. Give the formula for Newton's method for the function $f(x) = x^2 - 5$.

9–12. *Write the formula for Newton's method and use the given initial approximation to compute the approximations x_1 and x_2.*

9. $f(x) = x^2 - 6;\ x_0 = 3$
10. $f(x) = x^2 - 2x - 3;\ x_0 = 2$
11. $f(x) = 2 - \tan x;\ x_0 = 1$
12. $f(x) = x^3 - 2;\ x_0 = 2$

Practice Exercises

T 13–19. Finding roots with Newton's method *For the given function f and initial approximation x_0, use Newton's method to approximate a root of f. Stop calculating approximations when two successive approximations agree to five digits to the right of the decimal point after rounding. Show your work by making a table similar to that in Example 1.*

13. $f(x) = x^2 - 10;\ x_0 = 3$
14. $f(x) = x^3 + x^2 + 1;\ x_0 = -1.5$
15. $f(x) = \sin x + x - 1;\ x_0 = 0.5$
16. $f(x) = x^2 - \cos x;\ x_0 = 1.5$
17. $f(x) = \tan x - 2x;\ x_0 = 1.2$
18. $f(x) = 3 \sin x - 2;\ x_0 = 1$
19. $f(x) = x^2 - \sqrt{x} - 1;\ x_0 = 2$

T 20–26. Finding all roots *Use Newton's method to find all the roots of the following functions. Use preliminary analysis and graphing to determine good initial approximations.*

20. $f(x) = \cos x - \dfrac{x}{7}$
21. $f(x) = \cos 2x - x^2 + 2x$
22. $f(x) = \dfrac{x}{6} - \sec x$ on $[0, 8]$
23. $f(x) = 2 - \sin x - \sqrt{x}$
24. $f(x) = \dfrac{x^5}{5} - \dfrac{x^3}{4} - \dfrac{1}{20}$
25. $f(x) = \cos 3x + \sin 5x + x$
26. $f(x) = x^2(x - 100) + 1$

T 27–32. Finding intersection points *Use Newton's method to approximate all the intersection points of the following pairs of curves. Some preliminary graphing or analysis may help in choosing good initial approximations.*

27. $y = \sin x$ and $y = \dfrac{x}{2}$
28. $y = -x^3$ and $y = x + 1$
29. $y = \dfrac{1}{x}$ and $y = 4 - x^2$
30. $y = x^3$ and $y = x^2 + 1$
31. $y = 4\sqrt{x}$ and $y = x^2 + 1$
32. $y = \sin x$ and $y = \sqrt{x} - 1$

T 33. Retirement account Assume you invest \$10,000 at the end of each year for 30 years at an annual interest rate of r. The amount of money in your account after 30 years is $A = \dfrac{10{,}000((1 + r)^{30} - 1)}{r}$. Assume you want \$1,000,000 in your account after 30 years.
   a. Show that the minimum value of r required to meet your investment needs satisfies the equation $1{,}000{,}000r - 10{,}000(1 + r)^{30} + 10{,}000 = 0$.
   b. Apply Newton's method to solve the equation in part (a) to find the interest rate required to meet your investment goal.

T 34. Investment problem A one-time investment of \$2500 is deposited in a 5-year savings account paying a fixed annual interest rate r, with monthly compounding. The amount of money in the account after 5 years is $A(r) = 2500\left(1 + \dfrac{r}{12}\right)^{60}$.
   a. Use Newton's method to find the value of r if the goal is to have \$3200 in the account after 5 years.
   b. Verify your answer to part (a) algebraically.

T 35. Mortgage payments. The monthly payment on a \$200,000, 30-year (360-month) home loan is given by

$$m(r) = \frac{200{,}000(r/12)}{1 - (1 + r/12)^{-360}},$$

where r is the annual interest rate. Use Newton's method to determine the interest rate r that enables you to make monthly payments of \$1200 per month.

Section 4.9

6. Give the antiderivatives of $a \csc^2 x$, where a is a constant.

7. Give the antiderivatives of $1/x^2$.

8. Evaluate $\int a \cos x \, dx$ and $\int a \sin x \, dx$, where a is a constant.

9. If $F(x) = x^2 - 3x + C$ and $F(-1) = 4$, what is the value of C?

10. For a given function f, explain the steps used to solve the initial value problem $F'(t) = f(t)$, $F(0) = 10$.

Practice Exercises

11–22. Finding antiderivatives *Find all the antiderivatives of the following functions. Check your work by taking derivatives.*

11. $f(x) = 5x^4$

12. $g(x) = 11x^{10}$

13. $f(x) = 2 \sin x + 1$

14. $g(x) = -4 \cos x - x$

15. $p(x) = 3 \sec^2 x$

16. $q(s) = \csc^2 s$

17. $f(y) = -\dfrac{2}{y^3}$

18. $h(z) = -6z^{-7}$

19. $f(x) = \dfrac{1}{\sqrt{x}}$

20. $h(y) = \sqrt[4]{y}$

21. $f(x) = \dfrac{7}{2}x^{5/2}$

22. $f(t) = \pi$

23–54. Indefinite integrals *Determine the following indefinite integrals. Check your work by differentiation.*

23. $\int (3x^5 - 5x^9)\, dx$

24. $\int (3u^{-2} - 4u^2 + 1)\, du$

25. $\int \left(4\sqrt{x} - \dfrac{4}{\sqrt{x}}\right) dx$

26. $\int \left(\dfrac{5}{t^2} + 4t^2\right) dt$

27. $\int (5s + 3)^2\, ds$

28. $\int 5m(12m^3 - 10m)\, dm$

29. $\int (3x^{1/3} + 4x^{-1/3} + 6)\, dx$

30. $\int 5\sqrt[4]{x}\, dx$

31. $\int (3x + 1)(4 - x)\, dx$

32. $\int (4z^{1/3} - z^{-1/3})\, dz$

33. $\int \left(\dfrac{3}{x^4} + 2 - \dfrac{3}{x^2}\right) dx$

34. $\int \sqrt[5]{r^2}\, dr$

35. $\int \dfrac{4x^4 - 6x^2}{x}\, dx$

36. $\int \dfrac{12t^8 - t}{t^{3/2}}\, dt$

37. $\int \dfrac{x^2 - 36}{x - 6}\, dx$

38. $\int \dfrac{y^3 - 9y^2 + 20y}{y - 4}\, dy$

39. $\int (\csc^2 \theta + 2\theta^2 - 3\theta)\, d\theta$

40. $\int (\csc^2 \theta + 1)\, d\theta$

41. $\int \dfrac{2 + 3 \cos y}{\sin^2 y}\, dy$

42. $\int \sin t(4 \csc t - \cot t)\, dt$

43. $\int (\sec^2 x - 1)\, dx$

44. $\int \dfrac{\sec^3 v - \sec^2 v}{\sec v - 1}\, dv$

45. $\int (\sec^2 \theta + \sec \theta \tan \theta)\, d\theta$

46. $\int \dfrac{\sin \theta - 1}{\cos^2 \theta}\, d\theta$

47. $\int (3t^2 + 2 \csc^2 t)\, dt$

48. $\int \csc x(\cot x - \csc x)\, dx$

49. $\int \sec \theta(\tan \theta + \sec \theta + \cos \theta)\, d\theta$

50. $\int \dfrac{\csc^3 x + 1}{\csc x}\, dx$

51. $\int \dfrac{\tan x + \sec x}{\sec x}\, dx$

52. $\int \left(\sqrt[3]{x^2} + \sqrt{x^3}\right) dx$

53. $\int \sqrt{x}\left(2x^6 - 4\sqrt[3]{x}\right) dx$

54. $\int \dfrac{16 \cos^2 w - 81 \sin^2 w}{4 \cos w - 9 \sin w}\, dw$

55–60. Particular antiderivatives *For the following functions f, find the antiderivative F that satisfies the given condition.*

55. $f(x) = x^5 - 2x^2 + 1$; $F(0) = 1$

56. $f(x) = 4\sqrt{x} + 6$; $F(1) = 8$

57. $f(x) = 8x^3 + \sin x$; $F(0) = 2$

58. $f(t) = \sec^2 t$; $F(\pi/4) = 1$, $-\pi/2 < t < \pi/2$

59. $f(v) = \sec v \tan v$; $F(0) = 2$, $-\pi/2 < v < \pi/2$

60. $f(\theta) = 2 \sin \theta - 4 \cos \theta$; $F\left(\dfrac{\pi}{4}\right) = 2$

61–68. Solving initial value problems *Find the solution of the following initial value problems.*

61. $f'(x) = 2x - 3$; $f(0) = 4$

62. $g'(x) = 7x^6 - 4x^3 + 12$; $g(1) = 24$

63. $g'(x) = 7x\left(x^6 - \dfrac{1}{7}\right)$; $g(1) = 2$

64. $h'(t) = 1 + 6 \sin t$; $h\left(\dfrac{\pi}{3}\right) = -3$

65. $f'(u) = 4(\cos u - \sin u)$; $f\left(\dfrac{\pi}{2}\right) = 0$

66. $f'(x) = \sin x + \cos x + 1$; $f(\pi) = 3$

67. $y'(\theta) = \dfrac{\sqrt{2} \cos^3 \theta + 1}{\cos^2 \theta}$; $y\left(\dfrac{\pi}{4}\right) = 3$, $-\pi/2 < \theta < \pi/2$

68. $v'(x) = 4x^{1/3} + 2x^{-1/3}$; $v(8) = 40$, $x > 0$

T 69–72. Graphing general solutions *Graph several functions that satisfy each of the following differential equations. Then find and graph the particular function that satisfies the given initial condition.*

69. $f'(x) = 2x - 5$; $f(0) = 4$

70. $f'(x) = 3x^2 - 1$; $f(1) = 2$

71. $f'(x) = 3x + \sin x$; $f(0) = 3$

72. $f'(x) = \cos x - \sin x + 2$; $f(0) = 1$

73–78. Velocity to position *Given the following velocity functions of an object moving along a line, find the position function with the given initial position.*

73. $v(t) = 2t + 4$; $s(0) = 0$

74. $v(t) = 5$; $s(0) = 4$

75. $v(t) = 2\sqrt{t}$; $s(0) = 1$

76. $v(t) = 2 \cos t$; $s(0) = 0$

77. $v(t) = 6t^2 + 4t - 10$; $s(0) = 0$

78. $v(t) = 4t + \sin t$; $s(0) = 0$

79–84. Acceleration to position *Given the following acceleration functions of an object moving along a line, find the position function with the given initial velocity and position.*

79. $a(t) = -32$; $v(0) = 20$, $s(0) = 0$

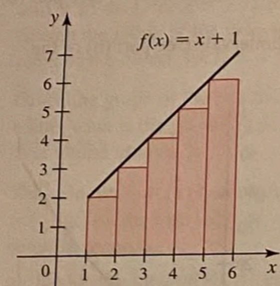
80. $a(t) = 4$; $v(0) = -3$, $s(0) = 2$

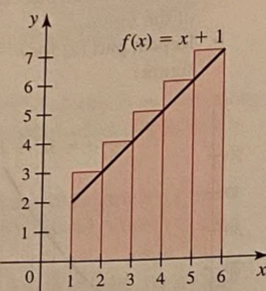
81. $a(t) = 0.2t$; $v(0) = 0$, $s(0) = 1$

Section 5.1

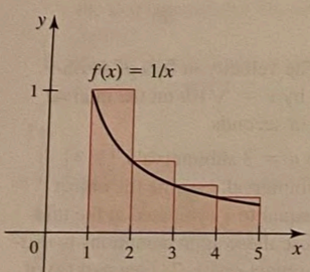
23–24. Left and right Riemann sums *Use the figures to calculate the left and right Riemann sums for f on the given interval and for the given value of n.*

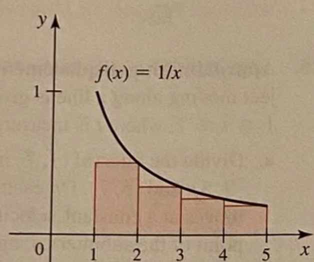
23. $f(x) = x + 1$ on $[1, 6]$; $n = 5$





24. $f(x) = \frac{1}{x}$ on $[1, 5]$; $n = 4$





25–32. Left and right Riemann sums *Complete the following steps for the given function, interval, and value of n.*

a. Sketch the graph of the function on the given interval.

b. Calculate Δx and the grid points $x_0, x_1, \ldots, x_n$.

c. Illustrate the left and right Riemann sums. Then determine which Riemann sum underestimates and which sum overestimates the area under the curve.

d. Calculate the left and right Riemann sums.

25. $f(x) = x + 1$ on $[0, 4]$; $n = 4$

26. $f(x) = 9 - x$ on $[3, 8]$; $n = 5$

T 27. $f(x) = \cos x$ on $\left[0, \frac{\pi}{2}\right]$; $n = 4$

28. $f(x) = \sin(\pi x/6)$ on $[0, 3]$; $n = 3$

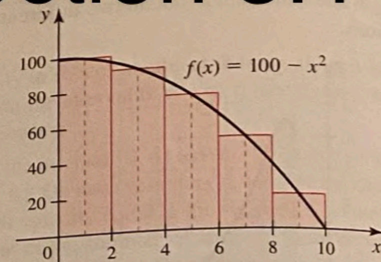
29. $f(x) = x^2 - 1$ on $[2, 4]$; $n = 4$

30. $f(x) = 2x^2$ on $[1, 6]$; $n = 5$

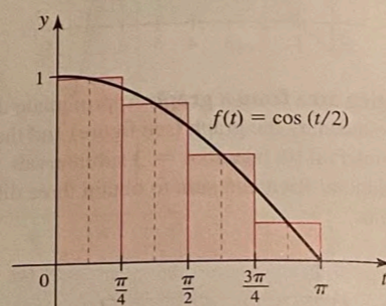
T 31. $f(x) = \sqrt{x}$ on $[0, 3]$; $n = 6$

T 32. $f(x) = 2^x$ on $[0, 1]$; $n = 4$

33. A midpoint Riemann sum Approximate the area of the region bounded by the graph of $f(x) = 100 - x^2$ and the x-axis on $[0, 10]$ with $n = 5$ subintervals. Use the midpoint of each subinterval to determine the height of each rectangle (see figure).



T 34. A midpoint Riemann sum Approximate the area of the region bounded by the graph of $f(t) = \cos \frac{t}{2}$ and the t-axis on $[0, \pi]$ with $n = 4$ subintervals. Use the midpoint of each subinterval to determine the height of each rectangle (see figure).

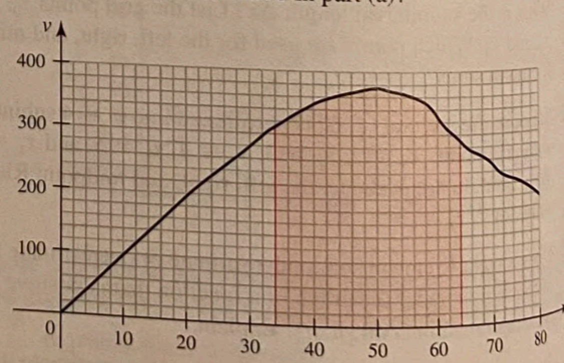


35. Free fall On October 14, 2012, Felix Baumgartner stepped off a balloon capsule at an altitude of almost 39 km above Earth's surface and began his free fall. His velocity in m/s during the fall is given in the figure. It is claimed that Felix reached the speed of sound 34 seconds into his fall and that he continued to fall at supersonic speed for 30 seconds. (*Source:* http://www.redbullstratos.com)

a. Divide the interval $[34, 64]$ into $n = 5$ subintervals with the grid points $x_0 = 34$, $x_1 = 40$, $x_2 = 46$, $x_3 = 52$, $x_4 = 58$, and $x_5 = 64$. Use left and right Riemann sums to estimate how far Felix fell while traveling at supersonic speed.

b. It is claimed that the actual distance that Felix fell at supersonic speed was approximately 10,485 m. Which estimate in part (a) produced the more accurate estimate?

c. How could you obtain more accurate estimates of the total distance fallen than those found in part (a)?



36. Free fall Use geometry and the figure given in Exercise 35 to estimate how far Felix fell in the first 20 seconds of his free fall.

37–42. Midpoint Riemann sums *Complete the following steps for the given function, interval, and value of n.*

a. Sketch the graph of the function on the given interval.
b. Calculate Δx and the grid points $x_0, x_1, \ldots, x_n$.
c. Illustrate the midpoint Riemann sum by sketching the appropriate rectangles.
d. Calculate the midpoint Riemann sum.

37. $f(x) = 2x + 1$ on $[0, 4]$; $n = 4$

T 38. $f(x) = 2\cos(\pi x/2)$ on $[0, 1]$; $n = 6$

T 39. $f(x) = \sqrt{x}$ on $[1, 3]$; $n = 4$

40. $f(x) = x^2$ on $[0, 4]$; $n = 4$

41. $f(x) = \frac{1}{x}$ on $[1, 6]$; $n = 5$

42. $f(x) = 4 - x$ on $[-1, 4]$; $n = 5$

43–44. Riemann sums from tables *Evaluate the left and right Riemann sums for f over the given interval for the given value of n.*

43. $[0, 2]$; $n = 4$

x	0	0.5	1	1.5	2
$f(x)$	5	3	2	1	1

44. $[1, 5]$; $n = 8$

x	1	1.5	2	2.5	3	3.5	4	4.5	5
$f(x)$	0	2	3	2	2	1	0	2	3

45. **Displacement from a table of velocities** The velocities (in mi/hr) of an automobile moving along a straight highway over a two-hour period are given in the following table.

t (hr)	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
v (mi/hr)	50	50	60	60	55	65	50	60	70

a. Sketch a smooth curve passing through the data points.
b. Find the midpoint Riemann sum approximation to the displacement on $[0, 2]$ with $n = 2$ and $n = 4$ subintervals.

46. **Displacement from a table of velocities** The velocities (in m/s) of an automobile moving along a straight freeway over a four-second period are given in the following table.

t (s)	0	0.5	1	1.5	2	2.5	3	3.5	4
v (m/s)	20	25	30	35	30	30	35	40	40

a. Sketch a smooth curve passing through the data points.
b. Find the midpoint Riemann sum approximation to the displacement on $[0, 4]$ with $n = 2$ and $n = 4$ subintervals.

47. **Sigma notation** Express the following sums using sigma notation. (Answers are not unique.)

a. $1 + 2 + 3 + 4 + 5$
b. $4 + 5 + 6 + 7 + 8 + 9$
c. $1^2 + 2^2 + 3^2 + 4^2$
d. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

48. **Sigma notation** Express the following sums using sigma notation. (Answers are not unique.)

a. $1 + 3 + 5 + 7 + \cdots + 99$
b. $4 + 9 + 14 + \cdots + 44$
c. $3 + 8 + 13 + \cdots + 63$
d. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{49 \cdot 50}$

49. **Sigma notation** Evaluate the following expressions.

a. $\sum_{k=1}^{10} k$
b. $\sum_{k=1}^{6} (2k + 1)$
c. $\sum_{k=1}^{4} k^2$
d. $\sum_{n=1}^{5} (1 + n^2)$
e. $\sum_{m=1}^{3} \frac{2m + 2}{3}$
f. $\sum_{j=1}^{3} (3j - 4)$
g. $\sum_{p=1}^{5} (2p + p^2)$
h. $\sum_{n=0}^{4} \sin \frac{n\pi}{2}$

T 50. **Evaluating sums** Evaluate the following expressions by two methods. (i) Use Theorem 5.1. (ii) Use a calculator.

a. $\sum_{k=1}^{45} k$
b. $\sum_{k=1}^{45} (5k - 1)$
c. $\sum_{k=1}^{75} 2k^2$
d. $\sum_{n=1}^{50} (1 + n^2)$
e. $\sum_{m=1}^{75} \frac{2m + 2}{3}$
f. $\sum_{j=1}^{20} (3j - 4)$
g. $\sum_{p=1}^{35} (2p + p^2)$
h. $\sum_{n=0}^{40} (n^2 + 3n - 1)$

T 51–54. Riemann sums for larger values of n *Complete the following steps for the given function f and interval.*

a. For the given value of n, use sigma notation to write the left, right, and midpoint Riemann sums. Then evaluate each sum using a calculator.
b. Based on the approximations found in part (a), estimate the area of the region bounded by the graph of f and the x-axis on the interval.

51. $f(x) = 3\sqrt{x}$ on $[0, 4]$; $n = 40$

52. $f(x) = x^2 + 1$ on $[-1, 1]$; $n = 50$

53. $f(x) = x^2 - 1$ on $[2, 5]$; $n = 75$

54. $f(x) = \cos 2x$ on $\left[0, \frac{\pi}{4}\right]$; $n = 60$

T 55–58. Approximating areas with a calculator *Use a calculator and right Riemann sums to approximate the area of the given region. Present your calculations in a table showing the approximations for $n = 10, 30, 60$, and 80 subintervals. Make a conjecture about the limit of Riemann sums as $n \to \infty$.*

55. The region bounded by the graph of $f(x) = 12 - 3x^2$ and the x-axis on the interval $[-1, 1]$.

56. The region bounded by the graph of $f(x) = 3x^2 + 1$ and the x-axis on the interval $[-1, 1]$.

57. The region bounded by the graph of $f(x) = \frac{1 - \cos x}{2}$ and the x-axis on the interval $[-\pi, \pi]$.

58. The region bounded by the graph of $f(x) = \sin x + \cos x$ and the x-axis on the interval $[0, \pi/2]$.

59. **Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.

a. Consider the linear function $f(x) = 2x + 5$ and the region bounded by its graph and the x-axis on the interval $[3, 6]$. Suppose the area of this region is approximated using midpoint