Math 241: Homework 10 Solutions

Graded Problems:
Section 4.9: 20, 26, 62
Section 5.1: 30d, 40d

Section 4.8

Problem 10. *Write the formula for Newton's method and use the given initial approximation to compute the approximations* x_1 *and* x_2*:*

$$f(x) = x^2 - 2x - 3; \quad x_0 = 2$$

Solution

$$f(x) = x^2 - 2x - 3 \Rightarrow f'(x) = 2x - 2$$

Using Newton's method we have:

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 2 - \frac{f(2)}{f'(2)} \\ &= 2 - \frac{(2)^2 - 2(2) - 3}{2(2) - 2} \\ &= 2 - \frac{-3}{2} \\ &= 2 + \frac{3}{2} \\ &= \boxed{\frac{7}{2}} \end{aligned}$$

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= \frac{7}{2} - \frac{f\left(\frac{7}{2}\right)}{f'\left(\frac{7}{2}\right)} \\ &= \frac{7}{2} - \frac{\left(\frac{7}{2}\right)^2 - 2\left(\frac{7}{2}\right) - 3}{2\left(\frac{7}{2}\right) - 2} \\ &= \frac{7}{2} - \frac{\frac{9}{4}}{5} \\ &= \frac{7}{2} - \frac{9}{4} \cdot \frac{1}{5} \\ &= \frac{7}{2} - \frac{9}{20} \\ &= \boxed{\frac{61}{20}} \end{aligned}$$

□

Section 4.9

Problem 14. *Find all antiderivatives of the following function. Check your work by taking derivatives:*

$$g(x) = -4\cos x - x$$

Solution

$$-4(\sin x) - \frac{x^{1+1}}{1+1} + C = \boxed{-4\sin x - \frac{x^2}{2} + C}$$

□

Problem 20. *Find all antiderivatives of the following function. Check your work by taking derivatives:*

$$h(y) = \sqrt[3]{y}$$

Solution

$$\sqrt[3]{y} = y^{1/3}$$

Taking the antiderivative we have

$$\frac{y^{1/3+1}}{1/3+1} + C = \frac{y^{4/3}}{4/3} + C = \boxed{\frac{3}{4}y^{4/3} + C}$$

□

Problem 24. *Determine the following indefinite integrals. Check your work by differentiation:*

$$\int \left(3u^{-2} - 4u^2 + 1\right)\, du$$

Solution

$$\begin{aligned}
\int \left(3u^{-2} - 4u^2 + 1\right)\, du &= \int \left(3u^{-2} - 4u^2 + u^0\right)\, du \\
&= 3\left(\frac{u^{-2+1}}{-2+1}\right) - 4\left(\frac{u^{2+1}}{2+1}\right) + \frac{u^{0+1}}{0+1} + C \\
&= 3\left(\frac{u^{-1}}{-1}\right) - 4\left(\frac{u^3}{3}\right) + u + C \\
&= \boxed{-\frac{3}{u} - \frac{4u^3}{3} + u + C}
\end{aligned}$$

□

Problem 26. *Determine the following indefinite integrals. Check your work by differentiation:*

$$\int \left(\frac{5}{t^2} + 4t^2\right)\, dt$$

Solution

$$\begin{aligned}
\int \left(\frac{5}{t^2} + 4t^2\right)\, dt &= \int \left(5t^{-2} + 4t^2\right)\, dt \\
&= 5\left(\frac{t^{-2+1}}{-2+1}\right) + 4\left(\frac{t^3}{3}\right) + C \\
&= 5\left(\frac{t^{-1}}{-1}\right) + \frac{4t^3}{3} + C \\
&= \boxed{-\frac{5}{t} + \frac{4t^3}{3} + C}
\end{aligned}$$

□

Problem 46. *Determine the following indefinite integrals. Check your work by differentiation:*

$$\int \frac{\sin\theta - 1}{\cos^2\theta}\, d\theta$$

Solution

$$\begin{aligned}
\int \frac{\sin\theta - 1}{\cos^2\theta}\, d\theta &= \int \left(\frac{\sin\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}\right)\, d\theta \\
&= \int \left(\frac{\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta} - \frac{1}{\cos^2\theta}\right)\, d\theta \\
&= \int \left(\tan\theta\sec\theta - \sec^2\theta\right)\, d\theta \\
&= \boxed{\sec\theta - \tan\theta + C}
\end{aligned}$$

□

Problem 62. *Find the solution of the following initial value problem:*

$$g'(x) = 7x^6 - 4x^3 + 12; \quad g(1) = 24$$

Solution Taking the integral of $g'(x)$ we have

$$\begin{aligned} g(x) &= \int (7x^6 - 4x^3 + 12)\ dx \\ &= 7\left(\frac{x^7}{7}\right) - 4\left(\frac{x^4}{4}\right) + 12x + C \\ &= x^7 - x^4 + 12x + C \end{aligned}$$

We are given that $g(1) = 24$ so

$$\begin{aligned} g(1) = 24 &\Leftrightarrow (1)^7 - (1)^4 + 12(1) + C = 24 \\ &\Leftrightarrow 1 - 1 + 12 + C = 24 \\ &\Leftrightarrow 12 + C = 24 \\ &\Leftrightarrow C = 12 \\ &\Leftrightarrow \boxed{g(x) = x^7 - x^4 + 12x + 12} \end{aligned}$$

□

Problem 80. *Given the following acceleration functions of an object moving along a line, find the position function with the given initial velocity and position.*

$$a(t) = 4; v(0) = -3, s(0) = 2$$

Solution

$$\begin{aligned} v(t) &= \int a(t)\ dt \\ &= \int 4\ dt \\ &= 4t + C \end{aligned}$$

$$\begin{aligned} s(t) &= \int v(t)\ dt \\ &= \int (4t + C)\ dt \\ &= 4\left(\frac{t^2}{2}\right) + Ct + D \\ &= 2t^2 + Ct + D \end{aligned}$$

Using the given information we have

$$\begin{aligned} v(0) = 3 &\Leftrightarrow 4(0) + C = -3 \\ &\Leftrightarrow C = -3 \\ &\Leftrightarrow s(t) = 2t^2 - 3t + D \end{aligned}$$

$$\begin{aligned} s(0) = 2 &\Leftrightarrow 2(0)^2 - 3(0) + D = 2 \\ &\Leftrightarrow D = 2 \\ &\Leftrightarrow \boxed{s(t) = 2t^2 - 3t + 2} \end{aligned}$$

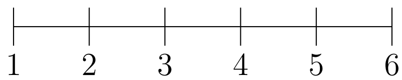
□

Section 5.1

Problem 30d. *Calculate the left and right Riemann sums:*

$$f(x) = 2x^2 \text{ on } [1, 6];\ n = 5$$

Solution Splitting the interval $[1, 6]$ into 5 pieces we have



Each rectangle has a width of 1.

$$\begin{aligned} L_5 &= 1f(1) + 1f(2) + 1f(3) + 1f(4) + 1f(5) \\ &= 2 + 8 + 18 + 32 + 50 \\ &= \boxed{110} \end{aligned}$$

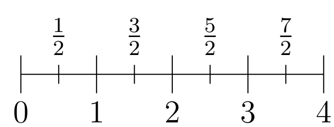
$$\begin{aligned} R_5 &= 1f(2) + 1f(3) + 1f(4) + 1f(5) + 1f(6) \\ &= 8 + 18 + 32 + 50 + 72 \\ &= \boxed{180} \end{aligned}$$

□

Problem 40d. *Calculate the midpoint Riemann sum:*

$$f(x) = x^2 \text{ on } [0, 4]; n = 4$$

Solution Splitting the interval $[0, 4]$ into 4 pieces and finding the midpoint of each piece we have



Each rectangle has a width of 1.

$$\begin{aligned} M_4 &= 1f\left(\frac{1}{2}\right) + 1f\left(\frac{3}{2}\right) + 1f\left(\frac{5}{2}\right) + 1f\left(\frac{7}{2}\right) \\ &= \frac{1}{4} + \frac{9}{4} + \frac{25}{4} + \frac{49}{4} \\ &= \frac{84}{4} \\ &= \boxed{21} \end{aligned}$$

□