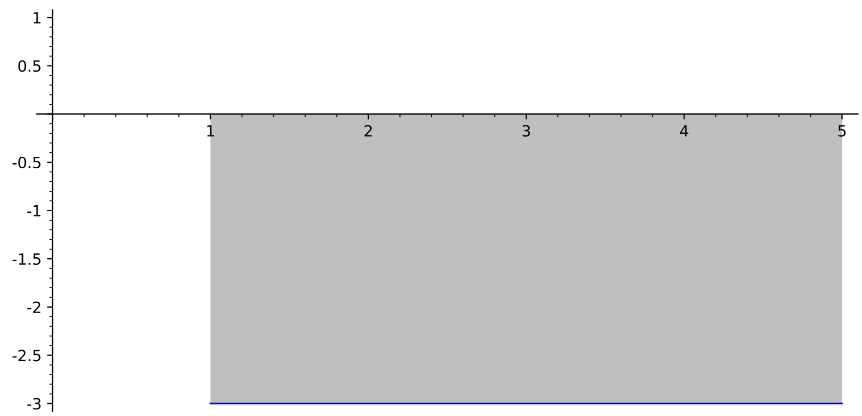
Math 241: Homework 11 Solutions

Graded Problems:
Section 5.2: 38
Section 5.3: 30, 36, 84
Section 5.4: 26

Section 5.2

Problem 8. *Sketch a graph of $y = -3$ on $[1, 5]$ and use geometry to find the exact value of $\int_1^5 (-3)\ dx$*

Solution



This is a rectangle of length 4 and height 3. Since it's under the x-axis, the area is negative. This gives an answer of $\boxed{-12}$

□

Problem 10. *Suppose $\int_1^3 f(x)\ dx = 10$ and $\int_1^3 g(x)\ dx = -20$. Evaluate $\int_1^3 (2f(x) - 4g(x))\ dx$ and $\int_3^1 (2f(x) - 4g(x))\ dx$.*

Solution

$$\begin{aligned}\int_1^3 (2f(x) - 4g(x))\ dx &= 2\int_1^3 f(x)\ dx - 4\int_1^3 g(x)\ dx \\ &= 2(10) - 4(-20) \\ &= 20 + 80 \\ &= \boxed{100}\end{aligned}$$

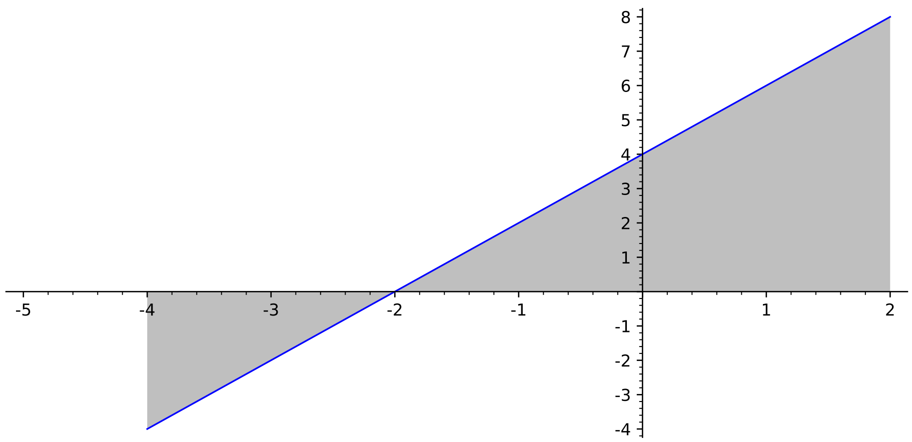
$$\begin{aligned}\int_3^1 (2f(x) - 4g(x)\ dx &= -\int_1^3 (2f(x) - 4g(x))\ dx \\ &= \boxed{-100}\end{aligned}$$

□

Problem 38. *Use geometry (not Riemann sums) to evaluate the following definite integrals. Sketch a graph of the integrant, show the region in question, and interpret your result.*

$$\int_{-4}^{2} (2x + 4)\ dx$$

Solution



The triangle on the left has area

$$-\frac{1}{2}bh = -\frac{1}{2}(2)(4) = -4$$

The triangle on the right has area

$$\frac{1}{2}bh = \frac{1}{2}(4)(8) = 16$$

This gives total area of

$$16 - 4 = \boxed{12}$$

□

Section 5.3

Problem 30. *Evaluate the following integrals using the Fundamental Theorem of Calculus:*

$$\int_0^2 (3x^2 + 2x)\ dx$$

Solution

$$\begin{aligned}
\int_0^2 (3x^2 + 2x)\ dx &= \left(3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2}\right)\Bigg|_0^2 \\
&= (x^3 + x^2)\big|_0^2 \\
&= (2^3 + 2^2) - (0^3 + 0^2) \\
&= 8 + 4 \\
&= \boxed{12}
\end{aligned}$$

□

Problem 34. *Evaluate the following integrals using the Fundamental Theorem of Calculus:*

$$\int_0^{\pi/4} 2\cos x\ dx$$

Solution

$$\begin{aligned}
\int_0^{\pi/4} 2\cos x\ dx &= 2\sin x\big|_0^{\pi/4} \\
&= 2\sin\frac{\pi}{4} - 2\sin 0 \\
&= 2 \cdot \frac{\sqrt{2}}{2} - 0 \\
&= \boxed{\sqrt{2}}
\end{aligned}$$

□

Problem 36. *Evaluate the following integrals using the Fundamental Theorem of Calculus:*

$$\int_4^9 \frac{2+\sqrt{t}}{\sqrt{t}}\ dt$$

Solution

$$\begin{aligned}
\int_4^9 \frac{2+\sqrt{t}}{\sqrt{t}}\ dt &= \int_4^9 \left(\frac{2}{\sqrt{t}} + \frac{\sqrt{t}}{\sqrt{t}}\right)\ dt \\
&= \int_4^9 (2t^{-1/2} + 1)\ dt \\
&= \left(2 \cdot \frac{t^{1/2}}{1/2} + t\right)\Bigg|_4^9 \\
&= (2 \cdot 2\sqrt{t} + t)|_4^9 \\
&= (4\sqrt{9} + 9) - (4\sqrt{4} + 4) \\
&= (12 + 9) - (8 + 4) \\
&= 21 - 12 \\
&= \boxed{9}
\end{aligned}$$

□

Problem 72. *Simplify the following expression:*

$$\frac{d}{dx}\int_0^x \sin^2 t\ dt$$

Solution $\boxed{\sin^2 x}$

□

Problem 84. *Simplify the following expression:*

$$\frac{d}{dx}\int_x^{x^2} \sin t^2 \ dt$$

Solution Let $u = x^2$

$$\begin{aligned}
\frac{d}{dx}\int_x^{x^2} \sin t^2 \ dt &= \frac{d}{dx}\left(\int_x^{0} \sin t^2 \ dt + \int_0^{x^2} \sin t^2 \ dt\right) \\
&= \frac{d}{dx}\left(-\int_0^{x} \sin t^2 \ dt + \int_0^{x^2} \sin t^2 \ dt\right) \\
&= -\frac{d}{dx}\left(\int_0^{x} \sin t^2 \ dt\right) + \frac{d}{dx}\left(\int_0^{x^2} \sin t^2 \ dt\right) \\
&= -\sin x^2 + \left(\frac{d}{du}\int_0^{u} \sin t^2 \ dt\right) \cdot \frac{du}{dx} \\
&= -\sin x^2 + \sin u^2 \cdot 2x \\
&= -\sin x^2 + 2x \sin\left(x^2\right)^2 \\
&= \boxed{-\sin x^2 + 2x \sin x^4}
\end{aligned}$$

□

Section 5.4

Problem 26. *Find the average value of the following functions on the given interval. Draw a graph of the function and indicate the average value.*

$$f(x) = x^2 + 1 \text{ on } [-2, 2]$$

Solution

$$\begin{aligned}
\text{Average Value} &= \frac{1}{2-(-2)} \int_{-2}^{2} (x^2+1)\ dx \\
&= \frac{1}{4} \int_{-2}^{2} (x^2+1)\ dx \\
&= \frac{1}{4} \left(\frac{x^3}{3} + x \right) \Big|_{-2}^{2} \\
&= \frac{1}{4} \left(\frac{8}{3} + 2 \right) - \frac{1}{4} \left(\frac{-8}{3} - 2) \right) \\
&= \frac{8}{12} + \frac{1}{2} + \frac{8}{12} + \frac{1}{2} \\
&= \frac{4}{6} + \frac{3}{6} + \frac{4}{6} + \frac{3}{6} \\
&= \frac{14}{6} \\
&= \boxed{\frac{7}{3}}
\end{aligned}$$

□

Problem 36. *A rock is launched vertically upward from the ground with a speed of 64 ft/s. The height of the rock (in ft) above the ground after t seconds is given by the function $s(t) = -16t^2 + 64t$. Find its average velocity during its flight.*

Solution First we have to find out how long its flight is

$$\begin{aligned}
s(t) = 0 &\Leftrightarrow -16t^2 + 64t = 0 \\
&\Leftrightarrow -16t(t-4) = 0 \\
&\Leftrightarrow t = 0, 4
\end{aligned}$$

So the flight is from $t = 0$ to $t = 4$

$$\begin{aligned}
\text{Average Velocity} &= \frac{s(4) - s(0)}{4 - 0} \\
&= \frac{0 - 0}{4} \\
&= \boxed{0}
\end{aligned}$$

□

Common Mistakes

- There are two things required to use FTC when taking the derivative of an integral. The first thing is that the lower bound has to be a number. If both bounds are variables, you need to split the integral first.

- Remember, there is NO SUCH THING as product rule or quotient rule for integrals. If there is a product/quotient, you need to simplify the integrand first before integrating. You CANNOT just integrate on thing and then the other and divide/multiply your results.

  For Example:

$$\int \frac{1+t}{t^3}\,dt = \int \left(\frac{1}{t^3} + \frac{t}{t^3}\right)\,dt = \int \left(t^{-3} + t^{-2}\right)\,dt$$

$$\int \frac{1+t}{t^3}\,dt \neq \frac{t + \frac{t^2}{2}}{\frac{t^4}{4}} + C$$