Math 241: Homework 12 Solutions

Graded Problems: All

Section 5.5

Problem 10. *Use the given substitution to evaluate the following indefinite integral. Check your answer by differentiation:*

$$\int (6x+1)\sqrt{3x^2+x}\ dx, \quad u = 3x^2 + x$$

Solution

$$u = 3x^2 + x \Rightarrow du = (6x+1)\ dx \Rightarrow dx = \frac{du}{6x+1}$$

$$\begin{aligned}
\int (6x+1)\sqrt{3x^2+x}\ dx &= \int \cancel{(6x+1)}\sqrt{u} \cdot \frac{du}{\cancel{6x+1}} \\
&= \int u^{1/2}\ du \\
&= \frac{u^{3/2}}{3/2} + C \\
&= \frac{2}{3}u^{3/2} + C \\
&= \boxed{\frac{2}{3}(3x^2+x)^{3/2} + C}
\end{aligned}$$

Check:

$$\frac{d}{dx}\left(\frac{2}{3}(3x^2+x)^{3/2} + C\right) = (3x^2+x)^{1/2}(6x+1) = (6x+1)\sqrt{3x^2+x}\checkmark$$

$\square$

Problem 24. *Use a change of variables to evaluate the following indefinite integral. Check your work by differentiating:*

$$\int \sin^{10}\theta \cos\theta \; d\theta$$

Solution

$$u = \sin\theta \Rightarrow du = \cos\theta \; d\theta \Rightarrow d\theta = \frac{du}{\cos\theta}$$

$$\begin{aligned}
\int \sin^{10}\theta \cos\theta \; d\theta &= \int u^{10} \cancel{\cos\theta} \cdot \frac{du}{\cancel{\cos\theta}} \\
&= \int u^{10} \; du \\
&= \frac{u^{11}}{11} + C \\
&= \boxed{\frac{\sin^{11}\theta}{11} + C}
\end{aligned}$$

Check:

$$\frac{d}{d\theta}\left(\frac{\sin^{11}\theta}{11} + C\right) = \sin^{10}\theta \cos\theta \checkmark$$

□

Problem 30. *Use a change of variables to evaluate the following indefinite integral. Check your work by differentiating:*

$$\int \sec 4w \tan 4w \; dw$$

Solution

$$u = 4w \Rightarrow du = 4 \; dw \Rightarrow dw = \frac{du}{4}$$

$$\begin{aligned}
\int \sec 4w \tan 4w \; dw &= \int \sec u \tan u \cdot \frac{du}{4} \\
&= \frac{1}{4}\int \sec u \tan u \; du \\
&= \frac{1}{4}\sec u + C \\
&= \boxed{\frac{1}{4}\sec 4w + C}
\end{aligned}$$

Check:

$$\frac{d}{dw}\left(\frac{1}{4}\sec 4w + C\right) = \frac{1}{4}\sec 4w \tan 4w \cdot 4 = \sec 4w \tan 4w \checkmark$$

□

Problem 44. *Use a change of variables to evaluate the following definite integral:*

$$\int_0^2 \frac{2x}{(x^2+1)^2}\ dx$$

Solution

$$u = x^2 + 1 \Rightarrow du = 2x\ dx \Rightarrow dx = \frac{du}{2x}$$

Changing the bounds:

$$x = 0 \Rightarrow u = 0^2 + 1 = 1 \text{ and } x = 2 \Rightarrow u = 2^2 + 1 = 5$$

$$\begin{aligned}
\int_0^2 \frac{2x}{(x^2+1)^2}\ dx &= \int_1^5 \frac{\cancel{2x}}{u^2} \cdot \frac{du}{\cancel{2x}} \\
&= \int_1^5 u^{-2}\ du \\
&= \frac{u^{-1}}{-1}\bigg|_1^5 \\
&= -\frac{1}{u}\bigg|_1^5 \\
&= -\frac{1}{5} - \left(-\frac{1}{1}\right) \\
&= -\frac{1}{5} + 1 \\
&= \boxed{\frac{4}{5}}
\end{aligned}$$

□

Problem 48. *Use a change of variables to evaluate the following definite integral:*

$$\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\ dx$$

Solution

$$u = \cos x \Rightarrow du = -\sin x\ dx \Rightarrow dx = -\frac{du}{\sin x}$$

Changing the bounds:

$$x = 0 \Rightarrow u = \cos 0 = 1 \text{ and } x = \frac{\pi}{4} \Rightarrow u = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\begin{aligned}
\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\ dx &= \int_1^{\sqrt{2}/2} \frac{\cancel{\sin x}}{u^2} \cdot -\frac{du}{\cancel{\sin x}} \\
&= -\int_1^{\sqrt{2}/2} u^{-2}\ du \\
&= -\frac{u^{-1}}{-1}\Bigg|_1^{\sqrt{2}/2} \\
&= \frac{1}{u}\Bigg|_1^{\sqrt{2}/2} \\
&= \frac{1}{\sqrt{2}/2} - \left(\frac{1}{1}\right) \\
&= \frac{2}{\sqrt{2}} - 1 \\
&= \boxed{\sqrt{2} - 1}
\end{aligned}$$

□