Math 241: Homework 1 Solutions

Graded Problems:
Section 1.1: 11, 64 Section 1.2: 16 Section 2.2: 18, 46 Section 2.5: 16

Section 1.1

Problem 11. *Let* $f(x) = 2x + 1$ *and* $g(x) = 1/(x-1)$. *Simplify the expressions* $f(g(1/2))$, $g(f(4))$, *and* $g(f(x))$.

Solution

$$\begin{aligned}
f\left(g\left(\frac{1}{2}\right)\right) &= f\left(\frac{1}{\frac{1}{2}-1}\right)\\
&= f\left(\frac{1}{-\frac{1}{2}}\right)\\
&= f(-2)\\
&= 2(-2)+1\\
&= -4+1\\
&= \boxed{-3}
\end{aligned}$$

$$\begin{aligned}
g(f(4)) &= g(2(4)+1)\\
&= g(9)\\
&= \frac{1}{9-1}\\
&= \boxed{\frac{1}{8}}
\end{aligned}$$

$$\begin{aligned}
g(f(x)) &= g(2x+1)\\
&= \frac{1}{(2x+1)-1}\\
&= \boxed{\frac{1}{2x}}
\end{aligned}$$

□

Problem 14. *If* $f(x) = \sqrt{x}$ *and* $g(x) = x^3 - 2$*, simplify the expressions* $(f \circ g)(3)$*,* $(f \circ f)(64)$*,* $(g \circ f)(x)$*, and* $(f \circ g)(x)$*.*

Solution

$$\begin{aligned}(f \circ g)(3) &= f(g(3)) \\ &= f((3)^3 - 2) \\ &= f(27 - 2) \\ &= f(25) \\ &= \boxed{5}\end{aligned}$$

$$\begin{aligned}(f \circ f)(64) &= f(f(64)) \\ &= f(\sqrt{64}) \\ &= f(8) \\ &= \sqrt{8} \\ &= \sqrt{4 \cdot 2} \\ &= \boxed{2\sqrt{2}}\end{aligned}$$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= (\sqrt{x})^3 - 2 \\ &= \boxed{x^{3/2} - 2}\end{aligned}$$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(x^3 - 2) \\ &= \boxed{\sqrt{x^3 - 2}}\end{aligned}$$

$\square$

Problem 64. *Simplify the difference quotient* $\frac{f(x+h)-f(x)}{h}$ *for the following function:*

$$f(x) = 4x - 3$$

Solution

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{[4(x+h)-3]-(4x-3)}{h} \\
&= \frac{(4x+4h-3)-(4x-3)}{h} \\
&= \frac{\cancel{4x}+4h-\cancel{3}-\cancel{4x}+\cancel{3}}{h} \\
&= \frac{4\cancel{h}}{\cancel{h}} \\
&= \boxed{4}
\end{aligned}$$

□

Section 1.2

Problem 16. *Find and graph the linear function that passes through the points* $(2, -3)$ *and* $(5, 0)$.

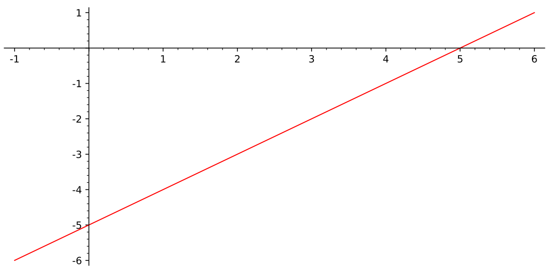
Solution

$$\begin{aligned} \text{Slope} &= \frac{-3-0}{2-5} \\ &= \frac{-3}{-3} \\ &= 1 \end{aligned}$$

Using point-slope form we have

$$y - 0 = 1(x-5) \Rightarrow \boxed{y = x - 5}$$

Note: Using the other point you would have $y + 3 = 1(x-2)$ which also simplifies to $y = x - 5$.



□

Problem 18. *Find the linear function whose graph passes through the point* $(-1, 4)$ *and is perpendicular to the line* $y = \frac{1}{4}x - 7$.

Solution Perpendicular lines have slopes that multiply to -1 (i.e. the slopes are opposite reciprocals). $y = \frac{1}{4}x - 7$ has slope $\frac{1}{4}$ so the perpendicular line has slope -4. Using point-slope form we have

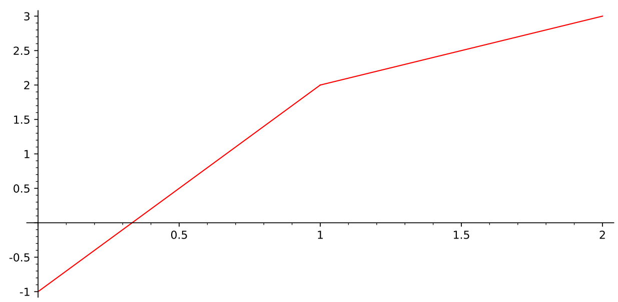
$$\boxed{y - 4 = -4(x+1)}$$

□

Problem 32. *Graph the following functions.*

$$f(x) = \begin{cases} 3x - 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$$

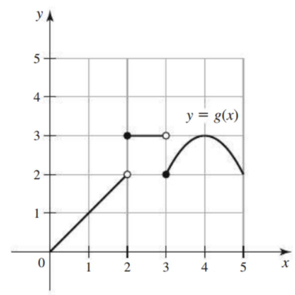
Solution



□

Section 2.2

Problem 18. *Use the graph of g in the figure to find the following values or state that they do not exist. If a limit does not exist, explain why.*



a) $g(2)$	*b)* $\lim_{x\to 2^-} g(x)$	*c)* $\lim_{x\to 2^+} g(x)$
d) $\lim_{x\to 2} g(x)$	*e)* $g(3)$	*f)* $\lim_{x\to 3^-} g(x)$
g) $\lim_{x\to 3^+} g(x)$	*h)* $g(4)$	*i)* $\lim_{x\to 4} g(x)$

Solution

a) 3	b) 2	c) 3
d) DNE, since the one-sided limits are not equal.	e) 2	f) 3
g) 2	h) 3	i) 3

□

Problem 34. *The Heaviside function is used in engineering applications to model flipping a switch. It is defined as*

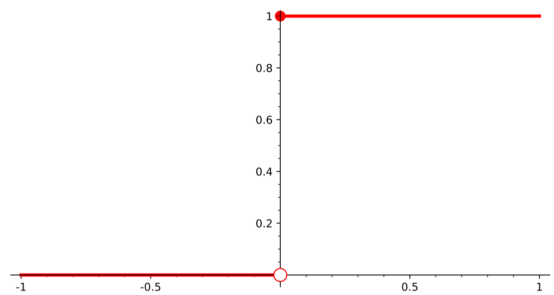
$$H(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$

a) Sketch a graph of h on the interval $[-1, 1]$*.*

b) Does $\lim_{x\to 0} H(x)$ *exist?*

Solution

a)



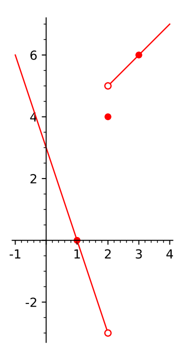
b) $\boxed{\text{no, } \lim_{x\to 0^-} H(x) = 0 \text{ and } \lim_{x\to 0^+} H(x) = 1}$

□

Problem 46. *Sketch the graph of a function with the given properties. You do not need to find a formula for the function.*

$$f(1) = 0, f(2) = 4, f(3) = 6, \lim_{x\to 2^-} f(x) = -3, \lim_{x\to 2^+} f(x) = 5$$

Solution



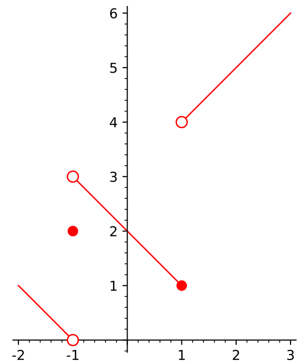
Note: There is more than one possible answer.

□

Problem 48. *Sketch the graph of a function with the given properties. You do not need to find a formula for the function.*

$$h(-1) = 2, \lim_{x \to -1^-} h(x) = 0, \lim_{x \to -1^+} h(x) = 3, h(1) = \lim_{x \to 1^-} h(x) = 1, \lim_{x \to 1^+} h(x) = 4$$

Solution

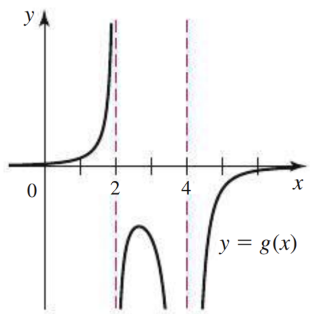


Note: There is more than one possible answer.

□

Section 2.4

Problem 8. *The graph of g in the figure has vertical asymptotes at $x = 2$ and $x = 4$. Analyze the following limits.*



a) $\lim_{x \to 2^-} g(x)$ *b)* $\lim_{x \to 2^+} g(x)$ *c)* $\lim_{x \to 2} g(x)$

d) $\lim_{x \to 4^-} g(x)$ *e)* $\lim_{x \to 4^+} g(x)$ *f)* $\lim_{x \to 4} g(x)$

Solution

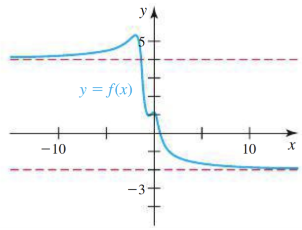
a) ∞ b) $-\infty$ c) DNE

d) $-\infty$ e) $-\infty$ f) $-\infty$

□

Section 2.5

Problem 2. *Evaluate* $\lim_{x\to\infty} f(x)$ *and* $\lim_{x\to-\infty} f(x)$ *using the figure.*

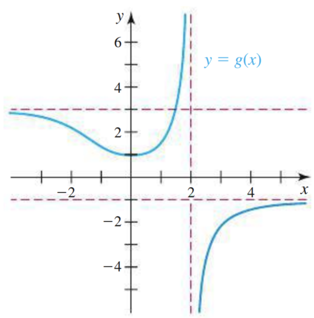


Solution $\boxed{\lim_{x\to\infty} f(x) = -2 \text{ and } \lim_{x\to-\infty} f(x) = 4}$

□

Problem 16. *The graph of* g *has a vertical asymptote at* $x = 2$ *and horizontal asymptotes at* $y = -1$ *and* $y = 3$ *(see figure). Determine the following limits:*

$$\lim_{x\to-\infty} g(x), \lim_{x\to\infty} g(x), \lim_{x\to2^-} g(x), \text{ and } \lim_{x\to2^+} g(x)$$



Solution

$$\boxed{\lim_{x\to-\infty} g(x) = 3, \lim_{x\to\infty} g(x) = -1, \lim_{x\to2^-} g(x) = \infty, \lim_{x\to2^+} g(x) = -\infty}$$

□

Common Mistakes

- Make sure your writing is clear. For example, many students wrote $\frac{1}{2}x$ instead of $\frac{1}{2x}$, which are not the same.
- Your work should be organized so that I understand which lines I should be reading next. Work that is written all over the paper is hard to read/grade.
- When you write an equal sign, the things on both sides MUST be equal. For example, for problem 64 in Section 1.1,

$$f(x+h) = \frac{4x+4h-3-4x+3}{h}$$

  is not mathematically correct.
- Find and graph means you have to (1) find the equation and (2) graph it. A lot of people just graphed it.
- Section 2.2 problem 18 said to explain why if a limit did not exist. Only one person did this so I didn't take off points this time but be sure to always read the instructions.