Math 241: Homework 2 Solutions

Graded Problems: All

Section 2.3

Problem 4. *Evaluate* $\lim_{x\to 4}\left(\frac{x^2-4x-1}{3x-1}\right)$

Solution

$$\begin{aligned}\lim_{x\to 4}\left(\frac{x^2-4x-1}{3x-1}\right) &= \frac{(4)^2-4(4)-1}{3(4)-1}\\ &= \frac{16-16-1}{12-1}\\ &= \boxed{\frac{-1}{11}}\end{aligned}$$

□

Problem 16. *Suppose*

$$f(x)=\begin{cases}4 & \text{if } x\le 3\\ x+2 & \text{if } x>3\end{cases}$$

Compute $\lim_{x\to 3^-} f(x)$ *and* $\lim_{x\to 3^+} f(x)$. *Then explain why* $\lim_{x\to 3} f(x)$ *does not exist.*

Solution

$$\begin{aligned}\lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-}(4)\\ &= \boxed{4}\end{aligned}$$

$$\begin{aligned}\lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+}(x+2)\\ &= 3+2\\ &= \boxed{5}\end{aligned}$$

Since $\lim_{x\to 3-} f(x) \neq \lim_{x\to 3^+} f(x)$ the two-sided limit $\lim_{x\to 3} f(x)$ does not exist.

□

Problem 34. *Find the following limits or state that they not exist. Assume a, b, c, and k are fixed real numbers.*

$$\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3}$$

Solution

$$\begin{aligned}
\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3} &= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} \\
&= \lim_{x\to 3}(x+1) \\
&= 3 + 1 \\
&= \boxed{4}
\end{aligned}$$

□

Problem 40. *Find the following limits or state that they not exist. Assume a, b, c, and k are fixed real numbers.*

$$\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h}$$

Solution

$$\begin{aligned}
\lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} &= \lim_{h\to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} \cdot \frac{5(5+h)}{5(5+h)} \\
&= \lim_{h\to 0} \frac{\frac{5\cancel{(5+h)}}{\cancel{5+h}} - \frac{\cancel{5}(5+h)}{\cancel{5}}}{5h(5+h)} \\
&= \lim_{h\to 0} \frac{5 - (5+h)}{5h(5+h)} \\
&= \lim_{h\to 0} \frac{\cancel{5} - \cancel{5} - h}{5h(5+h)} \\
&= \lim_{h\to 0} \frac{-\cancel{h}}{5\cancel{h}(5+h)} \\
&= \lim_{h\to 0} \frac{-1}{5(5+h)} \\
&= \frac{-1}{5(5+0)} \\
&= \boxed{-\frac{1}{25}}
\end{aligned}$$

□

Problem 56. *Find the following limits or state that they not exist. Assume a, b, c, and k are fixed real numbers.*

$$\lim_{x\to 1} \frac{x-1}{\sqrt{4x+5}-3}$$

Solution

$$\begin{aligned}
\lim_{x\to 1} \frac{x-1}{\sqrt{4x+5}-3} &= \lim_{x\to 1} \frac{x-1}{\sqrt{4x+5}-3} \cdot \frac{\sqrt{4x+5}+3}{\sqrt{4x+5}+3} \\
&= \lim_{x\to 1} \frac{(x-1)(\sqrt{4x+5}+3)}{(4x+5)-9} \\
&= \lim_{x\to 1} \frac{(x-1)(\sqrt{4x+5}+3)}{4x-4} \\
&= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{4x+5}+3)}{4\cancel{(x-1)}} \\
&= \lim_{x\to 1} \frac{\sqrt{4x+5}+3}{4} \\
&= \frac{\sqrt{4(1)+5}+3}{4} \\
&= \frac{\sqrt{9}+3}{4} \\
&= \frac{3+3}{4} \\
&= \frac{6}{4} \\
&= \boxed{\frac{3}{2}}
\end{aligned}$$

□

Problem 72. *Let*

$$g(x) = \begin{cases} 5x - 15 & \text{if } x < 4 \\ \sqrt{6x+1} & \text{if } x \geq 4 \end{cases}$$

Compute the following limits or state that they do not exist.

a) $\lim_{x \to 4^-} g(x)$ *b)* $\lim_{x \to 4^+} g(x)$ *c)* $\lim_{x \to 4} g(x)$

Solution

a)

$$\begin{aligned} \lim_{x \to 4^-} g(x) &= \lim_{x \to 4^-} (5x - 15) \\ &= 5(4) - 15 \\ &= 20 - 15 \\ &= \boxed{5} \end{aligned}$$

b)

$$\begin{aligned} \lim_{x \to 4^+} g(x) &= \lim_{x \to 4^+} \sqrt{6x+1} \\ &= \sqrt{6(4)+1} \\ &= \sqrt{25} \\ &= \boxed{5} \end{aligned}$$

c) $\boxed{5}$

□

Problem 74. *Let*

$$f(x) = \begin{cases} 0 & \text{if } x \le -5 \\ \sqrt{25 - x^2} & \text{if } -5 < x < 5 \\ 3x & \text{if } x \ge 5 \end{cases}$$

Compute the following limits or state that they do not exist.

a) $\lim_{x \to 5^-} f(x)$ *b)* $\lim_{x \to -5^+} f(x)$ *c)* $\lim_{x \to -5} f(x)$

d) $\lim_{x \to 5^-} f(x)$ *e)* $\lim_{x \to 5^+} f(x)$ *f)* $\lim_{x \to 5} f(x)$

Solution

a)

$$\begin{aligned} \lim_{x \to 5^-} f(x) &= \lim_{x \to 5^-} 0 \\ &= \boxed{0} \end{aligned}$$

b)

$$\begin{aligned} \lim_{x \to 5^+} f(x) &= \lim_{x \to 5^+} \sqrt{25 - x^2} \\ &= \sqrt{25 - (5)^2} \\ &= \sqrt{25 - 25} \\ &= \sqrt{0} \\ &= \boxed{0} \end{aligned}$$

c)

$$\boxed{0}$$

d)

$$\begin{aligned} \lim_{x \to 5^-} f(x) &= \lim_{x \to 5^-} \sqrt{25 - x^2} \\ &= \sqrt{25 - (5)^2} \\ &= \sqrt{0} \\ &= \boxed{0} \end{aligned}$$

e)

$$\begin{aligned} \lim_{x \to 5^+} f(x) &= \lim_{x \to 5^+} 3x \\ &= 3(5) \\ &= \boxed{15} \end{aligned}$$

f)

$$\boxed{\text{DNE}}$$

□

Common Mistakes

- You must carry the limit through the problem UNTIL you plug in. A lot of people are dropping the limit or carrying the limit too far.
- Remember to make sure that two things are equal when you write an equal sign between them. For example, a lot of you are dropping and adding limits between your steps.
- If a limit does not exist, do not write an equal sign. Just write DNE. This is because if you were to read it out loud, it should make sense.
  - $\lim_{x \to 1} f(x)$ DNE reads "the limit of f as x approaches 1 does not exist"
  - $\lim_{x \to 1} f(x) =$ DNE reads "the limit of f as x approaches 1 equals does not exist"