Math 241: Homework 6 Solutions

Graded Problems:
Section 3.8: 18
Section 3.9: 4, 16

Section 3.8

Problem 18. *Carry out the following steps.*

(a) *Use implicit differentiation to find* $\frac{dy}{dx}$
(b) *Find the slope of the curve at the given point.*

$$\sqrt{x} - 2\sqrt{y} = 0; \quad (4, 1)$$

Solution

(a)

$$\begin{aligned}
\sqrt{x} - 2\sqrt{y} = 0 &\Rightarrow x^{1/2} - 2y^{1/2} = 0 \\
&\Rightarrow \frac{1}{2}x^{-1/2} - y^{-1/2} \cdot \frac{dy}{dx} = 0 \\
&\Rightarrow y^{-1/2}\frac{dy}{dx} = \frac{1}{2}x^{-1/2} \\
&\Rightarrow \frac{1}{\sqrt{y}}\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \\
&\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cdot \sqrt{y} \\
&\Rightarrow \frac{dy}{dx} = \frac{\sqrt{y}}{2\sqrt{x}}
\end{aligned}$$

(b)

$$\frac{dy}{dx} = \frac{\sqrt{1}}{2\sqrt{4}} = \boxed{\frac{1}{4}}$$

□

Problem 26. *Carry out the following steps.*

(a) *Use implicit differentiation to find* $\dfrac{dy}{dx}$

(b) *Find the slope of the curve at the given point.*

$$(x+y)^{2/3} = y; \quad (4,4)$$

Solution

(a)

$$\begin{aligned}
(x+y)^{2/3} = y &\Rightarrow \frac{2}{3}(x+y)^{-1/3}\cdot\frac{d}{dx}(x+y) = \frac{dy}{dx}\\
&\Rightarrow \frac{2}{3}(x+y)^{-1/3}\left(1+\frac{dy}{dx}\right) = \frac{dy}{dx}\\
&\Rightarrow \frac{2}{3}(x+y)^{-1/3} + \frac{2}{3}(x+y)^{-1/3}\cdot\frac{dy}{dx} = \frac{dy}{dx}\\
&\Rightarrow \frac{2}{3}(x+y)^{-1/3} = \frac{dy}{dx} - \frac{2}{3}(x+y)^{-1/3}\frac{dy}{dx}\\
&\Rightarrow \frac{2}{3}(x+y)^{-1/3} = \frac{dy}{dx}\left(1-\frac{2}{3}(x+y)^{-1/3}\right)\\
&\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{3}(x+y)^{-1/3}}{1-\frac{2}{3}(x+y)^{-1/3}}\\
&\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{3}(x+y)^{-1/3}}{1-\frac{2}{3}(x+y)^{-1/3}}\cdot\frac{3}{3}\\
&\Rightarrow \frac{dy}{dx} = \boxed{\frac{2(x+y)^{-1/3}}{3-2(x+y)^{-1/3}}}
\end{aligned}$$

(b)

$$\begin{aligned}
\frac{dy}{dx} = \frac{2(4+4)^{-1/3}}{3-2(4+4)^{-1/3}} &\Rightarrow \frac{dy}{dx} = \frac{2(8)^{-1/3}}{3-2(8)^{-1/3}}\\
&\Rightarrow \frac{dy}{dx} = \frac{2\cdot\frac{1}{2}}{3-2\cdot\frac{1}{2}}\\
&\Rightarrow \frac{dy}{dx} = \frac{1}{3-1}\\
&\Rightarrow \frac{dy}{dx} = \boxed{\frac{1}{2}}
\end{aligned}$$

□

Section 3.9

Problem 4. *The temperature F in degrees Fahrenheit is related to the temperature C in degrees Celsius by the equation*

$$F = \frac{9}{5}C + 32$$

(a) Find an equation relating dF/dt to dC/dt.
(b) How fast is the temperature in an oven changing in degrees Fahrenheit per minute if it is rising at 10° Celsius per min?

Solution

(a)

$$F = \frac{9}{5}C + 32 \Rightarrow \boxed{\frac{dF}{dt} = \frac{9}{5}\frac{dC}{dt}}$$

(b)

$$\frac{dF}{dt} = \frac{9}{5}(10) = \frac{90}{5} = \boxed{18 \text{ degrees per min}}$$

□

Problem 10. *Assume $w = x^2y^4$, where x and y are functions of t. Find dw/dt when $x = 3$, $dx/dt = 2$, $dy/dt = 4$, and $y = 1$.*

Solution

$$w = x^2y^4 \Rightarrow \frac{dw}{dt} = \left(2x\frac{dx}{dt}\right)y^4 + x^2\left(4y^3\frac{dy}{dt}\right)$$

Plugging in the given numbers we have

$$\begin{aligned}\frac{dw}{dt} &= (2(3)(2))(1)^4 + (3)^2(4(1)^3(4)) \\ &= 12 + 9 \cdot 16 \\ &= 12 + 54 \\ &= \boxed{66}\end{aligned}$$

□

Problem 12. *The sides of a square decrease in length at a rate of 1 m/s.*

(a) At what rate is the area of the square changing when the sides are 5 m long?
(b) At what rate are the lengths of the diagonals of the square changing?

Solution

(a)

$$A = s^2 \Rightarrow \frac{dA}{dt} = 2s\frac{ds}{dt}$$

Plugging in $s = 5$ and $ds/dt = -1$ we have

$$\frac{dA}{dt} = 2(5)(-1) = \boxed{-10 \text{ m}^2/\text{s}}$$

(b) Using the pythagorean theorem, we have that the diagonal length is given by

$$D = s^2 + s^2 = 2s^2 \Rightarrow \frac{dD}{dt} = 4s\frac{ds}{dt}$$

Plugging in, we have

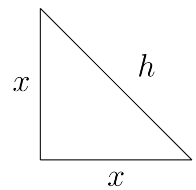
$$\frac{dD}{dt} = 4(5)(-1) = \boxed{-20 \text{ m/s}}$$

□

Problem 14. *The hypotenuse of an isosceles right triangle decreases in length at a rate of 4 m/s.*

(a) At what rate is the area of the triangle changing when the legs are 5 m long?
(b) At what rate are the lengths of the legs of the triangle changing?
(c) At what rate is the area of the triangle changing when the area is 4 m²?

Solution



We are given $dh/dt = -4$.

(a) We have:

$$A = \frac{1}{2}x^2 \text{ and } x^2 + x^2 = h^2 \Rightarrow x^2 = \frac{h^2}{2}$$

Putting this together we have

$$A = \frac{h^2}{4} \Rightarrow \frac{dA}{dt} = \frac{1}{2}h\frac{dh}{dt}$$

If $x = 5$ then $h^2 = 50 \Rightarrow h = \sqrt{50} = 5\sqrt{2}$. We now have

$$\frac{dA}{dt} = \frac{1}{2}(5\sqrt{2})(-4) = \boxed{-10\sqrt{2}\text{ m}^2/\text{s}}$$

(b)

$$A = \frac{1}{2}x^2 \Rightarrow \frac{dA}{dt} = x\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{dA/dt}{x}$$

Plugging in, we have

$$\frac{dx}{dt} = \frac{-10\sqrt{2}}{5} = \boxed{-2\sqrt{2}\text{ m/s}}$$

(c)

$$A = 4 \Rightarrow \frac{1}{2}x^2 = 4 \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$$

Solving for h we have

$$h^2 = 2(2\sqrt{2})^2 = 2(16) = 32$$

We have

$$\frac{dA}{dt} = \frac{1}{2}h\frac{dh}{dt} = \frac{1}{2}(32)(-4) = \boxed{-64\text{ m}^3/\text{s}}$$

□

Problem 16. *The edges of a cube increase at a rate of 2 cm/s. How fast is the volume changing when the length of each edge is 50 cm?*

Solution If the edge is x then the volume of the cube is

$$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

Plugging in $\frac{dx}{dt} = 2$ and $x = 50$, we have

$$\frac{dV}{dy} = 3(50)^2(2) = \boxed{15,000 \text{ cm}^3/\text{s}}$$

□

Common Mistakes

- Keep track of what equation you're using. A lot of people in Section 3.9 #4 were plugging into the wrong equation.
- Remember units!