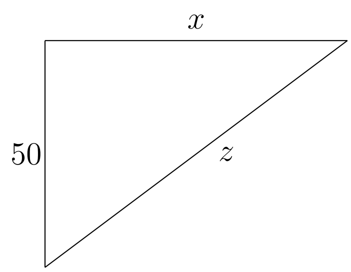
Math 241: Homework 7 Solutions

Graded Problems: All

Section 3.9

Problem 24. *Once Kate's kite reaches a height of 50 ft (above her hands), it rises no higher but drifts due east in a wind blowing 5 ft/s. How fast is the string running through Kate's hands at the moment when she has released 120 ft of string?*

Solution The picture looks like this



We are given that $\frac{dx}{dt} = 5$ and we want to find $\frac{dz}{dt}$ when $z = 120$ Using the pythagorean theorem we have

$$50^2 + x^2 = z^2 \Rightarrow 2x\frac{dx}{dt} = 2z\frac{dz}{dt}$$

When $z = 120$ we have

$$x^2 + 50^2 = 120^2 \Rightarrow x = \sqrt{11900} = 10\sqrt{119}$$

Plugging in everything we have

$$\begin{aligned}
2x\frac{dx}{dt} = 2z\frac{dz}{dt} &\Rightarrow 2(10\sqrt{119})(5) = 2(120)\frac{dz}{dt} \\
&\Rightarrow 100\sqrt{119} = 240\frac{dz}{dt} \\
&\Rightarrow \frac{dz}{dt} = \frac{100\sqrt{119}}{240} \\
&\Rightarrow \frac{dz}{dt} = \boxed{\frac{5\sqrt{119}}{12} \text{ ft/s}}
\end{aligned}$$

□

Problem 26. *A bug is moving along the right side of the parabola* $y = x^2$ *at a rate such that its distance from the origin is increasing at 1 cm/min.*

(a) At what rate is the x*-coordinate of the bug increasing at the point* $(2,4)$*?*
(b) Use the equation $y = x^2$ *to find an equation relating* $\frac{dy}{dt}$ *to* $\frac{dx}{dt}$
(c) At what rate is the y*-coordinate of the bug increasing at the point* $(2,4)$*?*

Solution

(a) We have from the pythagorean theorem that

$$D^2 = x^2 + y^2 \Rightarrow D^2 = x^2 + (x^2)^2 \Rightarrow D^2 = x^2 + x^4$$

Taking the derivative we have

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 4x^3\frac{dx}{dt}$$

At the point $(2,4)$, we have

$$2^2 + 4^2 = D^2 \Rightarrow D = \sqrt{20} = 2\sqrt{5}$$

Plugging in everything we have

$$\begin{aligned} 2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 4x^3\frac{dx}{dt} &\Rightarrow 2(2\sqrt{5})(1) = 2(2)\frac{dx}{dt} + 4(2)^3\frac{dx}{dt} \\ &\Rightarrow 4\sqrt{5} = 4\frac{dx}{dt} + 32\frac{dx}{dt} \\ &\Rightarrow 4\sqrt{5} = 36\frac{dx}{dt} \\ &\Rightarrow \frac{dx}{dt} = \frac{4\sqrt{5}}{36} \\ &\Rightarrow \frac{dx}{dt} = \boxed{\frac{\sqrt{5}}{9}} \end{aligned}$$

(b)

$$y = x^2 \Rightarrow \frac{dy}{dt} = 2x\frac{dx}{dt}$$

(c) Plugging in we have

$$\frac{dy}{dt} = 2(2)\cdot\frac{\sqrt{5}}{9} = \boxed{\frac{4\sqrt{5}}{9}}$$

□

Problem 31. *A water heated that has the shape of a right cylindrical tank with a radius of 1 ft and a height of 4 ft is being drained. How fast is water draining out of the tank (in ft^3/min) if the water level is dropping at 6 in/min*

Solution The volume of a cylinder is $V = \pi r^2 h$. Since the radius is 1ft and remains constant, we can plug it in to get $V = \pi h$. Taking the derivative we have

$$\frac{dV}{dt} = \pi \cdot \frac{dh}{dt}$$

Since $\frac{dh}{dt} = -6$ in/min, converting this to ft/min gives $-\frac{1}{2}$ ft/min. Plugging this in we have

$$\frac{dV}{dt} = \pi\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{2}\ \text{ft}^3/\text{min}}$$

□

Common Mistakes

- Keep track of what equation you're using. A lot of people in Section 3.9 #4 were plugging into the wrong equation.
- Remember units!