Midterm 1 – Math 241

Monday, March 2, 2020

This is a closed-book exam. No calculators allowed.

Justify your answers to obtain full credit (and partial credit, too).

You have the entire class time.

This exam consists of 6 questions.

Please verify that you have all pages.

If you need scratch paper, please ask.

Name:_____

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(1) Consider the function $f(x)$ given below. Estimate

(i) $\lim\limits_{x \to k^-} f(x)$

(ii) $\lim\limits_{x \to k^+} f(x)$

(iii) $\lim\limits_{x \to k} f(x)$

(iv) $f(k)$

(v) Is $f(x)$ continuous at k? (yes or no)

for each of the given values of k. If the given value does not exist, write "DNE", ∞, $-\infty$, or "undefined" as necessary:
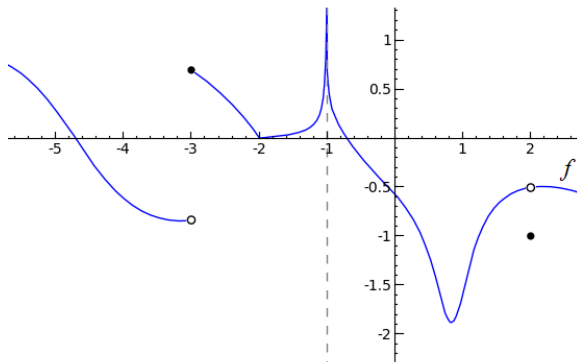

Figure 1: There is an asymptote at $x = -1$

(a) $k = -3$

(b) $k = -2$

(c) $k = -1$

(d) $k = 2$

(2) Evaluate the following limits. If the limit is ∞ or $-\infty$, please indicate as appropriate.

(a) $\displaystyle\lim_{x\to-2} \frac{x^2 + 5x + 6}{x^2 - 4}$

(b) $\displaystyle\lim_{x\to\infty} \frac{4x^3 - 21x^2 + 3x}{x^4 + 13x - 7}$

(c) $\displaystyle\lim_{x\to9} \frac{\sqrt{x} - 3}{x - 9}$

(d) $\displaystyle\lim_{x\to0} \frac{2x}{\sin 3x}$

(e) $\displaystyle\lim_{x\to\infty} \frac{\cos\left(x^2\right)}{x^4}$ (Hint: squeeze)

(3) Find the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(x) = 3x^2 + \dfrac{1}{x^2} - \sqrt{\pi}$

(b) $f(x) = \dfrac{8 + \sqrt[3]{x}}{3x - 5}$

(c) $f(x) = \sin x + x \tan x$

(d) $f(x) = (x^2 + 2x + 4)^{1/2}$

(4) Show that $4x^3 - 6x^2 - 6x + 5 = 0$ has at least one solution. Be sure to state any theorems you are using and why you can use them.

(5) Find an equation of the line tangent to $3x^2 + 2xy + y = 0$ at $(1, -1)$.

(6) A right cylindrical tank is filled with water and is being **drained** at a rate of 25 cm^3/sec. The tank stands upright and has radius 10 cm (the radius is constant). How fast is the height of the water in the tank dropping? The volume of a right cylinder is given by $V = \pi r^2 h$.

Use the **limit definition** of a derivative to differentiate $f(x) = x^2 + x + 4$. You get no points if you differentiate it directly using properties of derivatives.

Question	Score
1	
2	
3	
4	
5	
6	
Bonus	
Total	