Math 241 Midterm 1 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures and homework problems.

Previous Semester's Exam Problems

(1) Evaluate the following limits. Be as specific as possible (i.e write ∞ or $-\infty$ instead of DNE when applicable):

(a) $\lim_{x\to -1} \dfrac{x^2 - 5x - 6}{x+1}$

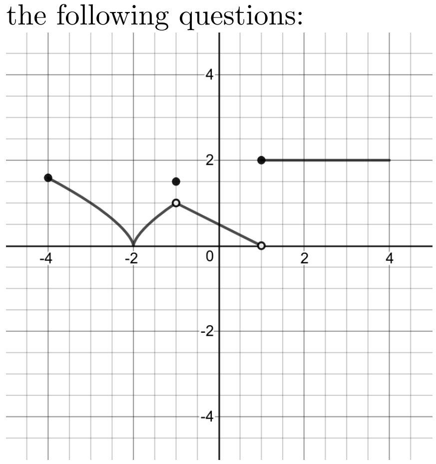
(b) $\lim_{x\to 3^-} \dfrac{4}{(x-3)^2}$

(c) $\lim_{x\to \infty} \dfrac{x^3 + x^2 + 2}{7x^3 + x + 1}$

(d) $\lim_{x\to 25} \dfrac{x - 25}{\sqrt{x} - 5}$

(e) $\lim_{x\to 0} \dfrac{\sin 4x}{5x}$

(2) The function $f(x)$ is defined for $-4 \le x \le 4$ and is graphed below. Use the graph to answer the following questions:



(a) What is $\lim_{x\to -1} f(x)$?

(b) What is $\lim_{x\to 1} f(x)$?

(c) Give the intervals where $f(x)$ is continuous, be careful to include the endpoints if necessary.

(d) Does the function appear to be differentiable at $x = -2$? Explain why or why not.

(3) Consider the function $f(x)$ given below. Find

(i) $\lim_{x\to k^-} f(x)$

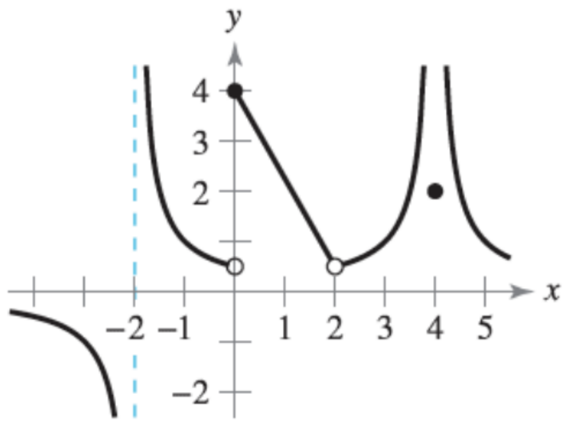
(ii) $\lim_{x\to k^+} f(x)$

(iii) $\lim_{x \to k} f(x)$

(iv) $f(k)$

(v) Is $f(x)$ continuous at k? (yes or no)

for each of the given values of k. If the given value does not exist, write "DNE", ∞, $-\infty$, or "undefined" as necessary:



(a) $k = -1$

(b) $k = 0$

(c) $k = 2$

(d) $k = 4$

(4) On which interval(s) is the following function continuous. Justify your answers using limits:

$$f(x) = \begin{cases} 1 - x^2 & x < -1 \\ 1 + x & -1 \leq x \leq 1 \\ -3 & x > 1 \end{cases}$$

(5) Evaluate $\lim_{x \to 0} (x^2 \sin(4x) + 1)$

(6) Show that the equation $x^3 - x^2 + 2x - 7 = 0$ has a solution in the interval $[1, 2]$. State any theorems you use to support your answer.

(7) Let $f(x) = 5 + x - x^4$. Use the intermediate value theorem to show that there is at least one point where $f(x) = 0$.

(8) (a) Using the limit definition of a derivative, differentiate the following:

$$f(x) = x^2 - 3x - 1$$

(b) Find the equation of the line tangent to $f(x)$ at $x = 1$

(9) Differentiate the following functions. You do not need to simplify your answers.

(a) $y = 3x \sin x$

(b) $y = \tan\left(x^2 + 1\right)$

(c) $y = \dfrac{x+1}{x^2+2}$

(d) $f(x) = 12x^2 - \dfrac{5}{\sqrt{x}} + 78$

(e) $y = \dfrac{\sin^2 x}{2x^3 + 4x^2 + 7}$

(10) Calculate the following derivatives:

(a)

$$y = \frac{x^3+1}{2-x}, \qquad \frac{dy}{dx} =$$

(b)

$$h(t) = \cos^2(\pi t) + 3, \qquad h'(t) =$$

(c)

$$f(x) = \sin(x)(x^2+3) + x^{4/3}, \qquad f'(x) =$$

(d)

$$y^4 - \sqrt{x} + 2y = 2, \qquad y' \text{ at } (1,1)$$

(11) Find an equation of the tangent line to $x^2 + xy + y^2 = 3$ at $(1,1)$

(12) Let a particle's motion be given by $s(t) = \sqrt{t - \cos t}$ for $t > 1$.

(a) Find the particle's **velocity** and **acceleration** as functions of t.

(b) What is the particle's speed at $t = 3\pi/2$?

(13) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius **decreases** 1/4 in. per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 in. The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

(14) A person leaves a given point and travels north at 3 mph. Another person leaves the same point at the same time and travels east at 4 mph. At what rate is the distance between the two people changing at the instant when they have traveled 2 hours?

Extra Practice Problems

(1) Calculate the following limits.

(a) $\lim_{t\to 1} \dfrac{t^2+t-2}{t^2-1}$

(b) $\lim_{x\to 2} \dfrac{\sqrt{x^2+12}-4}{x-2}$

(c) $\lim_{x\to\infty} \dfrac{7x^3}{x^3-3x^2+6x}$

(d) $\lim_{x\to 0} \dfrac{1}{x^3-1}+1$

(e) $\lim_{x\to 1} \dfrac{3x-4}{x^2+x+1}$

(f) $\lim_{x\to -1} \dfrac{x^2+x-2}{x^3+1}$

(g) $\lim_{x\to -1^-} \dfrac{|x+1|}{x+1}$

(h) $\lim_{x\to 1} \dfrac{x^2-1}{x-1}$

(i) $\lim_{x\to\infty} \dfrac{2x^3-4x^2+5x}{17x^2+1}$

(j) $\lim_{x\to -\infty} \dfrac{3x^2+4}{x+7}$

(k) $\lim_{t\to 2^-} \dfrac{t+2}{t-2}$

(l) $\lim_{x\to 0} \dfrac{\sin x}{3x}$

(m) $\lim_{x\to 0} x\cos\left(\dfrac{1}{x}\right)$

(2) Calculate the following limits. State clearly any theorems that you use.

(a)

$$\lim_{x\to 3^-} \frac{|x-3|}{(6-2x)}$$

(b)

$$\lim_{t\to 0} \frac{\sin(3t)}{t}$$

(c)

$$\lim_{h\to 1} \frac{\sqrt{h+8}-3}{1-h}$$

(d)

$$\lim_{x\to 0} x^2\cos\left(\frac{3}{x}\right)-11$$

(3) Describe on which intervals the following functions are continuous (show your work):

(a) $y = \dfrac{\sin x}{x-2}$

(b) $f(x) = \begin{cases} 3-x, & x<2 \\ \frac{x}{2}+1, & 2\le x<4 \\ 3, & x\ge 4 \end{cases}$

(c) $f(x) = \begin{cases} 1-x^2 & x<-1 \\ 1+x & -1\le x\le 1 \\ -3 & x>1 \end{cases}$

(4) Show that the equation $x^3-15x+1=0$ has at least three solutions in the interval $[-4,4]$

(5) (a) **Use the definition of derivative** to show that the derivative of $f(x)=x^2-x$ at $x=-2$ is -5, i.e. $f'(-2)=-5$.

(b) Find an equation for the tangent line to $f(x)=x^2-x$ at $x=-2$.

(6) Use the definition of the derivative (limit definition) to find the derivatives of the following:

(a) $f(x) = \sqrt{x}$
(b) $f(x) = x^2 - x$
(c) $f(x) = \frac{1}{x}$

(7) Consider the function $f(x) = 5 - x^2$.

(a) Find the equation for the secant line to the graph of $f(x)$ that passes through the points $(1,4)$ and $(2,1)$.
(b) Find $f'(x)$ using the definition of a derivative.
(c) Find the equation for the tangent line to the graph of $f(x)$ at the point $(1,4)$.
(d) Find the equation for the tangent line to the graph of $f(x)$ at the point $(2,1)$.

(8) A particle is moving along the x-axis. Its position at time t is given by the function $s(t) = -2t^2 + 5t - 2$.

(a) Find the particle's average velocity v_{av} between $t = 1$ and $t = 4$.
(b) Find the particle's instantaneous velocity at $t = 1$.

(9) If a particle's motion is given by the equation $s(t) = 4t^3 - 10t^2 + 5$, find its velocity and acceleration as functions of t. What is its speed at $t = 1$

(10) Find the first derivatives of the following:

(a) $y = 6x^2 - 10x - 5x^{-2}$
(b) $y = x^2 \sin x + 2x \cos x - 2 \sin x$
(c) $h(x) = x \tan\left(2\sqrt{x}\right) + 7$
(d) $y = \dfrac{\cot x}{1 + \cot\left(x^2 + x\right)}$
(e) $y = \left(1 - \dfrac{x}{7}\right)^{-7}$

(11) Find equations for the tangent and normal lines to $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$ at $(-1,0)$.

(12) The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When the length is 12cm and the width is 5cm, find the rates of change of **a)** the area, **b)** the perimeter, and **c)** the lengths of the diagonals of the rectangle. Which of these quantities are decreasing, and which are increasing?

(13) A child flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from them at a rate of 25 ft/sec. How fast must they let out the string when the kite is 500 ft away from them?