(1) Show that $x^3 + 6x + 3 = 0$ has exactly one solution in the interval $[-3, 0]$. Make sure you name what theorems you are using and why you can use them.

(2) Consider the function $f(x) = 9x - \frac{1}{3}x^3$

(a) Find the intervals where f is increasing and the intervals where f is decreasing.

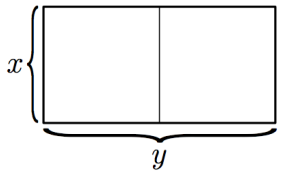
(b) Find both coordinates of any local max and local min of f.

(c) Find the intervals where f is concave up, and the intervals where f is concave down.

(d) Find both coordinates of any inflection point(s) of f.

(e) Graph the function

(3) A 24 m^2 rectangular farm is to be enclosed by a fence and divided into two equal parts by another fence (picture below). What dimensions will require the smallest total length of fence? How much fence will be needed?



(4) An object is thrown upward off a cliff that is 25m tall ($s(0) = 25$) with an initial velocity of 20 m/s ($v(0) = 20$). The acceleration of the object is given by

$$a(t) = -10 \text{ m/s}^2$$

(a) Find the equation for the position $s(t)$.

(b) When does the object reach its maximum height?

(c) When does the object hit the bottom ground?

(5) Calculate the following integrals. Simplify your answers if possible.

(a) $\int_0^1 (x^2 + \sqrt{x})$

(b) $\int \left(\frac{1}{x^3} - 1 + 2\sec^2 x \right) \, dx$

(c) $\int \cos^5(2x) \sin(2x) \, dx$

(d) $\int_0^1 t^3(1 + t^4)^3 \, dt$

(6) Find the following derivative:

$$\frac{d}{dx} \int_0^{x^3} \sin t \, dt$$

Optional Extra Credit: Solve Problem 6 (correctly lol) using two different methods.