Math 241 Midterm 2 Review Problems

These problems are intended to help you prepare for the test. Test problems will be similar to, but not the same as, the problems below. *This list of problems is not all inclusive; it does not represent every possible type of problem.* It is suggested that you review lectures and homework problems.

Previous Semester's Exam Problems

(1) Show that $x^4 - 4x = 1$ has exactly one solution on $[-1, 0]$. Please state explicitly any theorems and how you are using them.

(2) Show that $f(x) = 2x^3 + 3x^2 + 6x + 1$ has exactly one real root in $[-1, 0]$. Be sure to state and explain any theorems that you use.

(3) Let $f(x) = x^3 + 3x^2$

(a) Find the (open) intervals where f is increasing and where f is decreasing.

(b) Find all relative extrema (both x and y coordinates). Indicate whether it is a relative maximum or relative minimum.

(c) Find the (open) intervals where f is concave up and where f is concave down

(d) Find all inflection point(s) (both x and y coordinates)

(e) Using the information from parts (a)-(d), graph the function. Label all relative extrema and inflection point(s).

(4) Consider the function $f(x) = x^3 - 6x^2 + 9x$

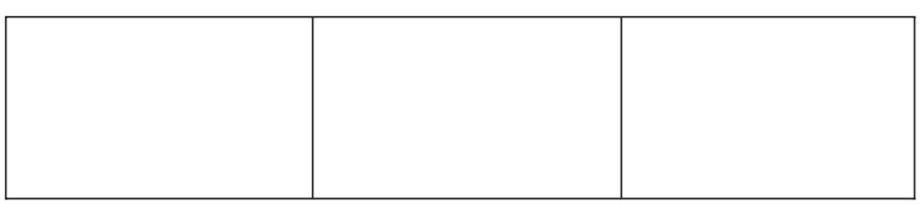
(a) Find the open intervals where f is increasing and the intervals where f is decreasing.

(b) Find both coordinates of any local extrema of the graph of f.

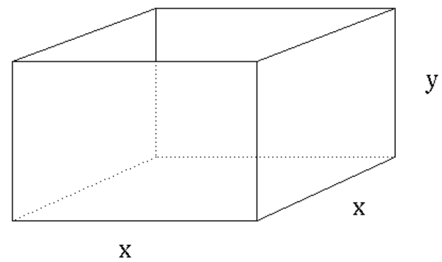
(c) Find the intervals where f is concave up, and the intervals where f is concave down.

(d) Find the both coordinates of any inflection point(s) of f.

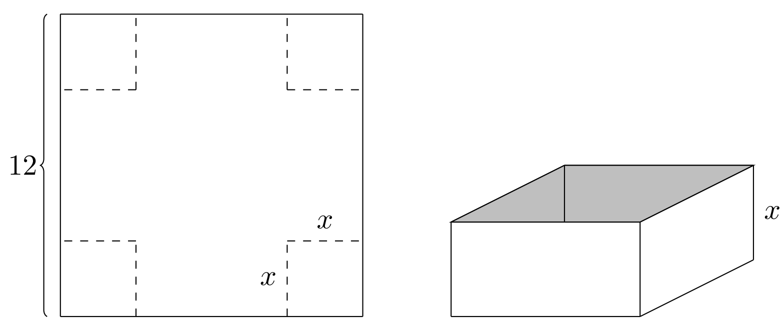
(5) An ecologist is conducting a research project on breeding pheasants in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the sketch. Find the **dimensions** of the pen that has the maximum area she can enclose with 3600 m of fencing.



(6) A box with a square base must have a volume of 8 in^3. What are the dimensions of the box that will minimize the amount of material needed to build it (i.e. minimize surface area).



(7) A box with no top is constructed by cutting equal-sized squares from the corners of a 12 cm by 12 cm sheet of metal and bending up the sides. What is the largest possible volume of such a box? See the pictures below. (Note: The domain of x is $(0, 6)$.)



(8) Use mid-points to approximate the area above the x-axis and under $x^2 + 6$ from $x = 0$ to $x = 6$ using 3 rectangles.

(9) A particle's acceleration is given by $a(t) = 6t + 2$. Its velocity at 1 sec is -1 m/s. Its initial position is given by $s(0) = 5$. Find the position function $s(t)$.

(10) Solve the initial value problem $\dfrac{dy}{dx} = 9x^2 - 4x + 5$, $y(-1) = 0$

(11) A ball is thrown from a cliff that is 6 feet from the ground ($s(0) = 6$) with initial velocity 100ft/sec ($v(0) = 100$). If the acceleration due to gravity is -32 ft/sec^2 ($a(t) = -32$), find the equation $s(t)$ for the position of the ball at time t.

(12) Find the following integrals:

(a) $\displaystyle\int_0^4 2(\sqrt{t} - t)\ dt$

(b) $\displaystyle\int \frac{1 + 2t^3}{t^3}\ dt$

(c) $\displaystyle\int \tan^4 x \sec^2 x dx$

(d) $\displaystyle\int_0^\pi 2 \sin x \cos^2 x$

(e) $\displaystyle\int \frac{x}{(x^2 + 2)^3}$

(13) Calculate the following integrals.

(a) $\int \left(\dfrac{x^2 + 7x^5 + 5}{x^2} \right) \, dx$

(b) $\int \tan^4 x \sec^2 x \, dx$

(c) $\int_1^2 (x^2 + 3x - 1) \, dx$

(d) $\int_0^4 \dfrac{x}{\sqrt{x^2 + 9}} \, dx$

Extra Practice Problems

(1) Find the absolute maximum and minimum values of the following functions of the given intervals.

  (a) $f(x) = x^2 - 1, \; -1 \le x \le 2$

  (b) $f(x) = \sqrt[3]{x}, \; -1 \le x \le 8$

(2) Explain why $g(t) = \sqrt{t} + \sqrt{1+t} - 4$ has exactly one solution in the interval $(0, \infty)$. State any theorems used.

(3) For the following functions, **a)** find the critical points, **b)** classify them as local maxima, local minima, or neither, **c)** find where the function is increasing, **d)** find where the function is concave up, and **e)** sketch the graph.

  (a) $y = x^4 - 2x^2$

  (b) $y = x^5 - 5x^4$

(4) A rectangular plot of land will be bounded on one side by a river and on the other three sides by some sort of fence. With 800 m of fencing at your disposal, what is the largest area you can enclose, and what are its dimensions?

(5) Suppose you want to build a steel box with an open top and square base. Find the dimensions for a box of volume 500 ft^3 that will weigh as little as possible.

(6) Find the dimensions of a right circular cylinder of maximum volume that can be inscribed in a sphere of radius 10 cm. What is the maximum volume?

(7) Use Newton's method to find the positive fourth root of 2 by solving the equation $x^4 - 2 = 0$. Start with $x_0 = 1$ and find x_2.

(8) Find the most general antiderivative for the following. Check your answer by differentiation.

  (a) $f(x) = \dfrac{1}{x^2} - x^2 - \dfrac{1}{3}$

  (b) $f(x) = 2x(1 - x^{-3})$

(9) Solve the following initial value problems.

  (a) $\dfrac{dr}{d\theta} = -\pi \sin \pi\theta, \; r(0) = 0$

  (b) $\dfrac{d^3y}{dx^3} = 6; \; y''(0) = -8, \; y'(0) = 0, \; y(0) = 5$

(10) The acceleration of an object is given by $\dfrac{3t}{8}$ find the position given that $v(4) = 3$ and $s(4) = 4$.

(11) Using 4 rectangles of equal length and the following rules find Riemann sums estimates for $f(x) = -x^2 + 16$ from $x = -2$ to $x = 2$.

  (a) Left-hand endpoints

  (b) Right-hand endpoints

  (c) Midpoints

(12) Find $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t\ dt$

(a) by evaluating the integral and differentiating the result.

(b) by differentiating the integral directly

(13) Evaluate the following integrals

(a) $\int \tan x \sec^2 x\ dx$

(b) $\int \frac{x}{\sqrt{4x^2+9}}\ dx$

(c) $\int \sec^2(5x)\ dx$

(d) $\int x(2x+1)^5\ dx$

(e) $\int_0^2 x\sqrt{x^2+1}\ dx$

(f) $\int_0^{\sqrt{\pi}/2} x \sin x^2\ dx$

(g) $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right)\ dx$

(h) $\int x^{-3}(x+1)\ dx$

(i) $\int_0^{\pi/3} 2\sec^2 x\ dx$

(j) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\ dx$

(k) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\ dt$

(l) $\int_{-1}^{1} t^3(1+t^4)^3\ dt$

(m) $\int_0^{\pi/6} (1-\cos 3t)\sin 3t\ dt$