2.3

Measures of Central Tendency

A measure of central tendency is a value that represents a typical, or central, entry of a data set. The three most common measures:

Mean

The sum of the data entries divided by the number of entries.

Denoted **μ** for population mean and **$\bar{x}$** for sample mean

$$\mu = \frac{\sum x}{N} \qquad \bar{x} = \frac{\sum x}{n}$$

Median

The value that lies in the middle of the data set when it is ordered numerically.

If there are two numbers in the middle, average them.

Mode

The data entry (or entries) that occur the most frequently.

If all entries have equal frequency, the data set has no mode.

Example:

274, 235,
223, 268,
290, 285, 235

Consider the data set given to the left describing the weights (in pounds) of a sample of adults. Find the mean, median, and mode.

Mean ≈ 258.6
Median: 268
Mode: 235

General rounding guidelines: Round your final answer to one more decimal place than the original entries. Do not do any rounding until you get to the final calculation

Example: The prices (in dollars) of a sample of roundtrip flight tickets are: 872, 432, 397, 427, 388, 782, 397, 782

Find the mean, median, and mode of the data

Mean ≈ 559.6
Median: 429.5
Modes: 397 and 782

Definition: If a data set has two modes, it is called __bimodal__

All three measures of central tendency are supposed to describe what the “typical” entry of a data set looks like, but sometimes some are more useful/accurate than others.

One thing that can cause this is an <u>outlier</u>, i.e. a data entry that is very different from the other entries.

Outliers can occur naturally or due to some kind of error

Example: The following data consists of the ages of students in class sample:

20	20	20	20	20
20	21	21	21	21
22	22	22	22	23
23	23	24	24	65

Find and compare the mean, median, and mode.

Weighted Mean

A weighted mean is the mean of a data set whose entries have varying weights (e.g. GPA, class grade, etc.)

$$\bar{x} = \frac{\sum xw}{\sum w}$$

where *w* is the weight of each entry *x*

Example: If A=4, B=3, C=2, D=1, and F=0, determine the following student's GPA (weighted mean)

Grade	Credit Hours
C	3
C	4
D	1
A	3
C	2
B	3

$$\bar{x} = \frac{2(3) + 2(4) + 1(1) + 4(3) + 2(2) + 3(3)}{3 + 4 + 1 + 3 + 2 + 3}$$
$$= 2.5$$

Frequency Distribution Mean

The mean of a frequency distribution for a sample can be calculated with

$$\bar{x} = \frac{\sum xf}{n}$$

where *x* is the class midpoint and *f* is the class frequency

Example: Calculate the mean of the following frequency distribution

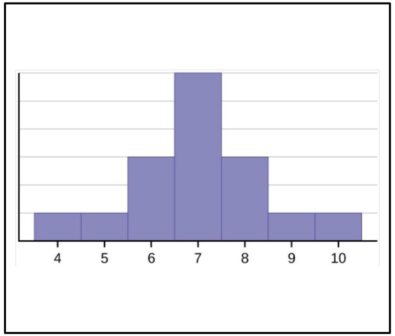
Class	Frequency
7-18	6
19-30	10
31-42	13
43-54	8
55-66	5
67-78	6
79-90	2

Note: The table does not give the class midpoints, so you first have to calculate those on your own.

Remember, the midpoint of a class can be found by averaging the lower and the upper limit.

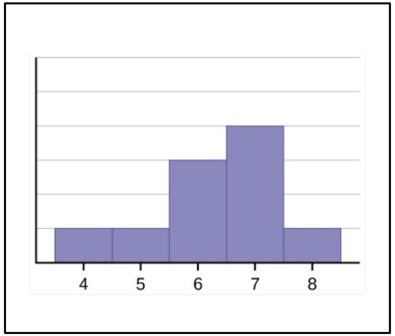
$$\begin{aligned}\bar{x} &= \frac{12.5(6) + 24.5(10) + 36.5(13) + 48.5(8) + 60.5(5) + 72.5(6) + 84.5(2)}{50} \\ &= \frac{2089}{5} \\ &= 41.78\end{aligned}$$

Types of Frequency Distribution Shapes



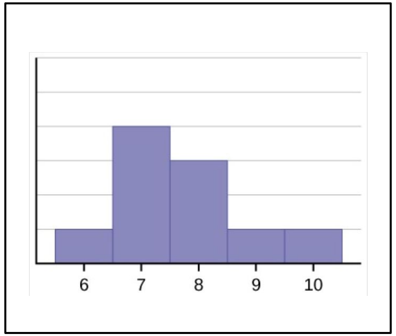
Symmetric

If it is unimodal (has one mode), the mean, median, and mode are equal



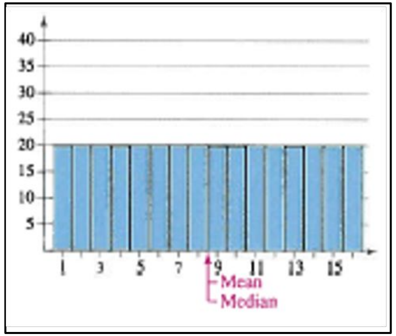
Skewed Left (negatively skewed)

The mean is less than the median. The median is usually less than the mode.



Skewed Right (positively skewed)

The mean is greater than the median. The median is usually greater than the mode.



Uniform (regular)

The mean and median are equal. There is no mode.