2.5

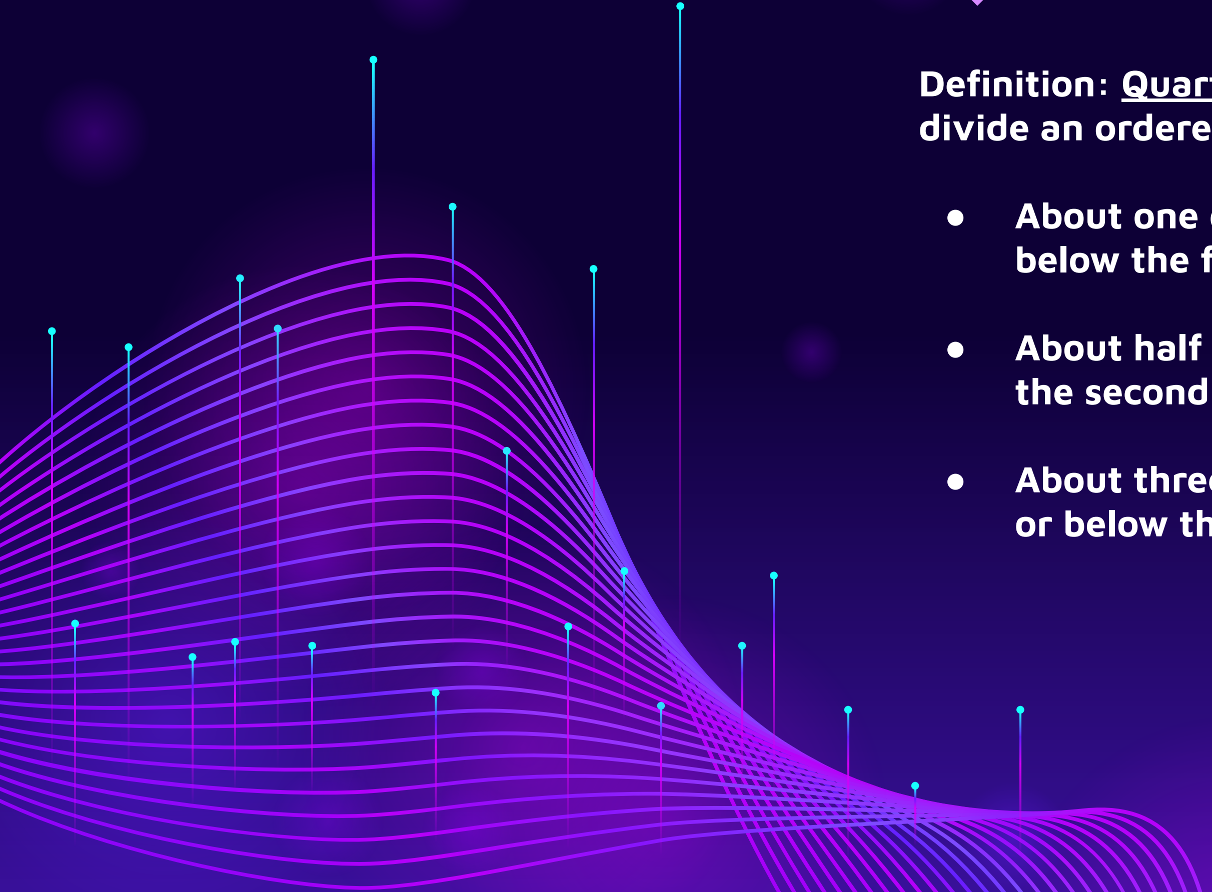
Measures of Position

Definition:

Fractiles (also called quantiles) are numbers that partition (i.e. divide) an ordered data set into equal parts

For example, the median is a fractile that splits an ordered data set into two equal parts

Quartiles



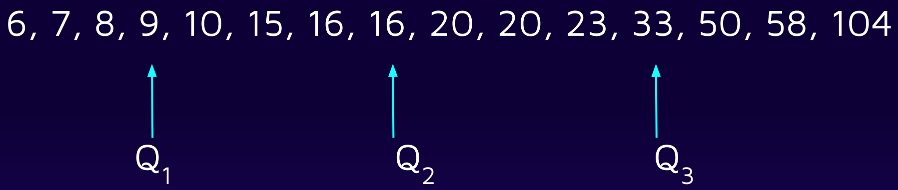
Definition: **<u>Quartiles</u>** (Q_1,Q_2,Q_3) approximately divide an ordered set into four equal parts

- About one quarter of the data fall on or below the first quartile (Q_1)
- About half of the data fall on or below the second quartile (Q_2 aka the median)
- About three quarters of the data fall on or below the third quartile (Q_3)

Example: The number of nuclear power plants in the top 15 nuclear power-producing countries in the world are listed. Find the first, second, and third quartiles of the data set.

7	**20**	**16**
6	**58**	**9**
20	**50**	**23**
33	**8**	**10**
15	**16**	**104**

First we need to put these in order:



6, 7, 8, 9, 10, 15, 16, 16, 20, 20, 23, 33, 50, 58, 104

Note: You can also use a TI-84 or other graphing calculator to find the quartiles. Input the quantities using STAT and then hit STAT–CALC–1: 1-Var Stats. Scroll down.

Definition: The Interquartile Range (IQR) of a data set gives you the range of the middle 50% of the data

$$\text{IQR} = Q_3 - Q_1$$

You can identify outliers using the IQR:

1. Find Q_1 and Q_3
2. Find the IQR using the above formula
3. Any data between Q_1-1.5(IQR) and Q_3+1.5(IQR) is considered normal. Anything outside of that is considered an outlier.

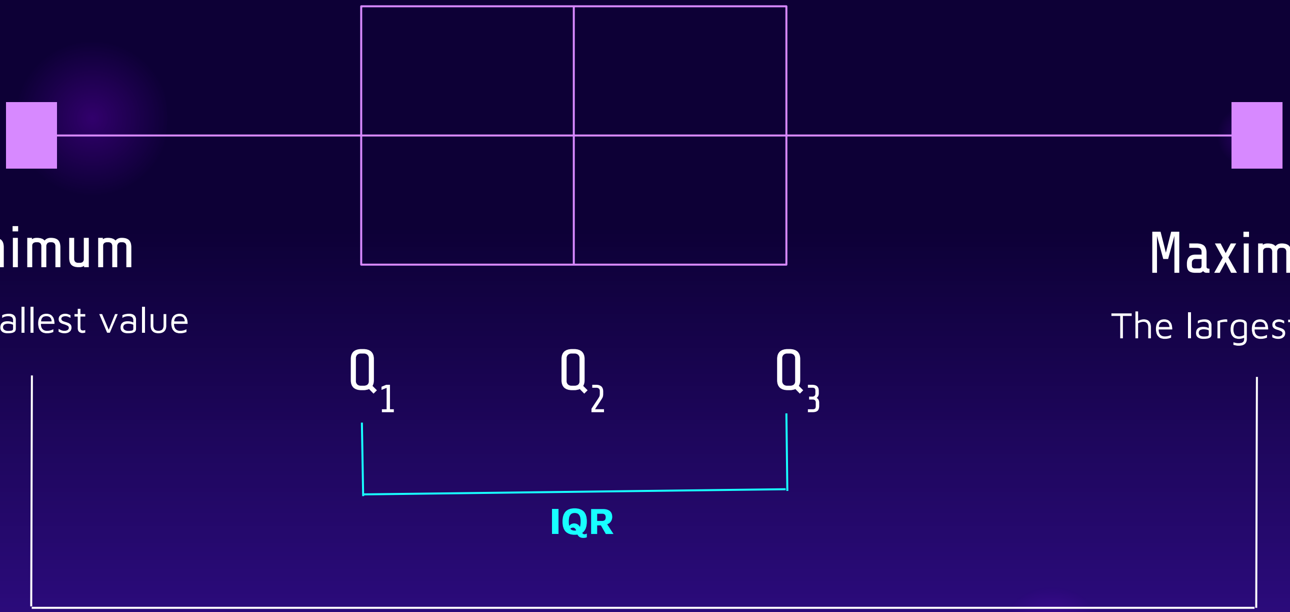
Example: Nuclear data

We have $Q_1 = 9$ and $Q_3 = 33$ with

Normal values lie between -27 and 69, so 104 is considered an outlier

A box-and-whisker plot (or boxplot)

A tool that shows a 5-number summary of a data set: the minimum, Q_1, Q_2, Q_3, and the maximum



Minimum

The smallest value

Maximum

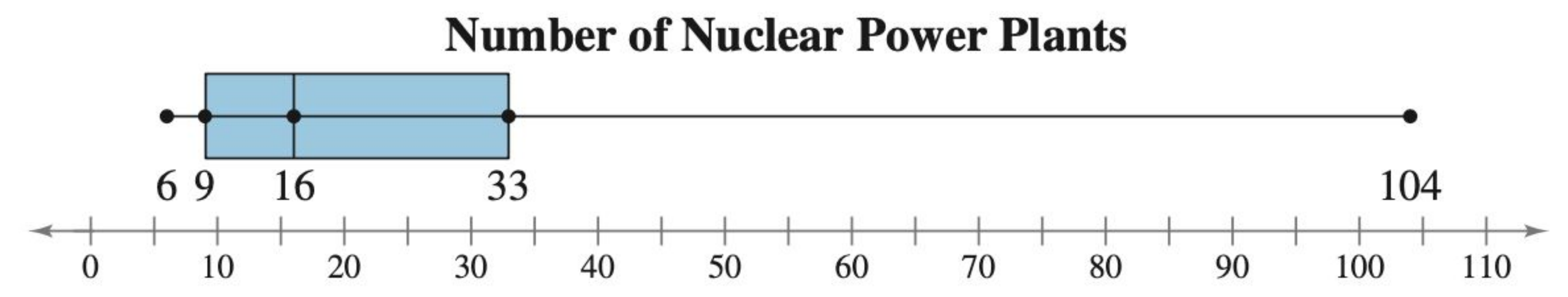
The largest value

Q_1 Q_2 Q_3

IQR

Range

Example: Nuclear data

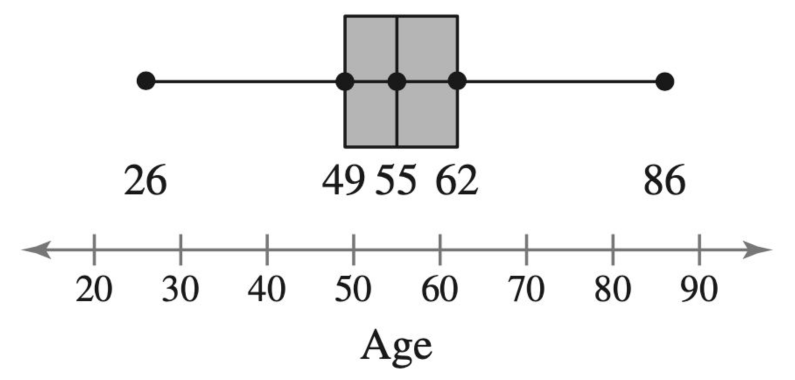


About 50% of the data lies within the box (between 9 and 33), approximately 35% of the data are less than 9 and approximately 25% are greater than 33.

The shape also indicates that the distribution is skewed right

Example: The data on the left indicates the ages of 50 women. Draw a boxplot and interpret the results

Ages				
26	31	35	37	43
43	43	44	45	47
48	48	49	50	51
51	51	51	52	54
54	54	54	55	55
55	56	57	57	57
58	58	58	58	59
59	59	62	62	63
64	65	65	65	66
66	67	67	72	86



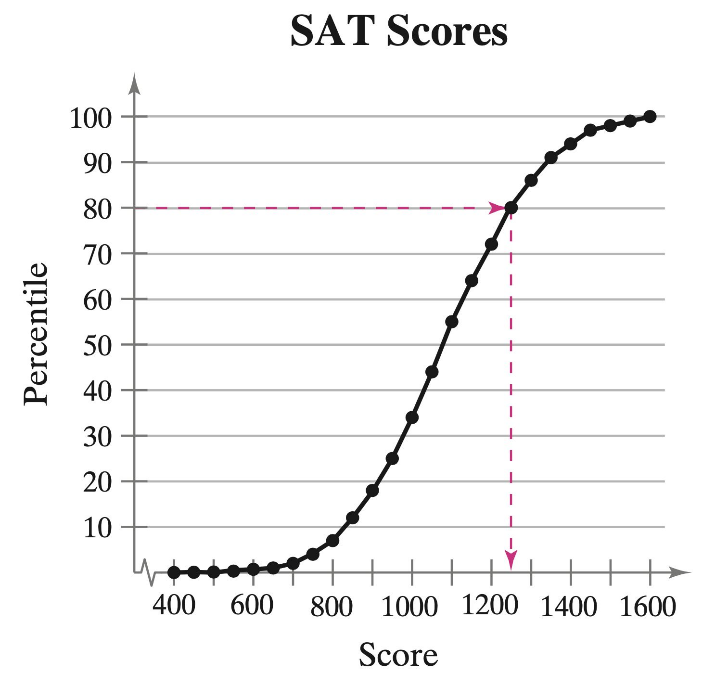
About 50% of the women are between 49 and 62 years old. 25% are less than 49 years old and 25% are greater than 62 years old

Other Fractiles

Deciles	Splits a data set into 10 equal pieces. Notation for this is: $D_1, D_2, D_3, \ldots, D_9$
Percentiles	Splits a data set into 100 equal pieces. Notation for this is: $P_1, P_2, P_3, \ldots, P_{99}$ This is often used in health and education. The nth percentile indicates that n% of the population falls at or below that number

Example: A sixth month old baby weighing in the 78th percentile means that 78% of all sixth month old babies weigh the same or less

Example: What score corresponds to the 80th percentile? Interpret the results.



The 80th percentile lies around an SAT score of 1250.

This means that approximately 80% of students score 1250 or less on their SATs

To find the percentile of a data value *x* :

$$\text{percentile} = \frac{\text{number of entries less than } x}{\text{total number of entries}} \cdot 100$$

Definition:

The standard score (or *z*-score) of a data value *x* represents the number of standard deviations *x* lies from the mean μ

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

A negative *z*-score indicates that *x* is less than the mean, a positive *z*-score indicates *x* is greater than the mean, and a *z*-score of 0 indicates that x is equal to the mean

A value with a *z* score less than -2 or greater than 2 is considered unusual

Example:

Assume there are two groups of people, Group A and Group B.

For Group A, the average height is given by $\mu=69.9$ in with standard deviation $\sigma=3.0$ in.

For Group B, the average height is given by $\mu=64.3$ in with standard deviation $\sigma=2.6$ in.

Compare and interpret the z-scores for a 6 foot person in Group A vs Group B