

Chapter 3

Probability

ACSTA101 - Professor MG

3.1

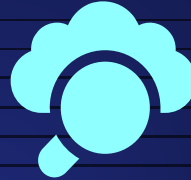
Basic Concepts of Probability and Counting

Definitions



Experiment

A probability experiment is an action or trial through which specific results are obtained



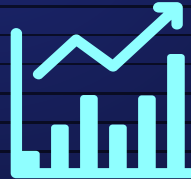
Outcome

An outcome is a result of a probability experiment



Sample Space

The set of all possible outcomes is called the sample space



Event

An event is a subset of the sample space that consists of one or more outcomes.



Example

A survey asked people for their blood types. Determine how many outcomes there are and identify the sample space.

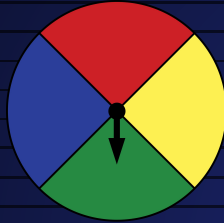
There are 4 main types (A, B, AB, and O) that can be positive or negative. This gives 8 outcomes.

Sample space = $\{O+, O-, A+, A-, B+, B-, AB+, AB-\}$



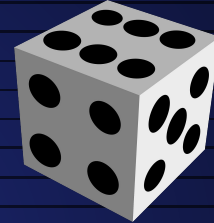


More Examples



**Probability
Experiment: Spinning
the spinner**

4 possible outcomes.
Sample space = {red, yellow,
blue, green}



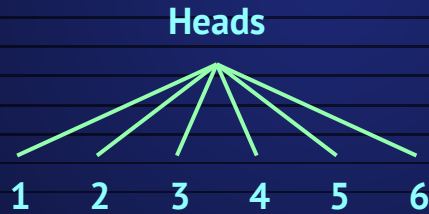
**Probability
experiment: Rolling a
6-sided die**

6 possible outcomes.
Sample space = {1, 2, 3, 4, 5, 6}



More Complex Example

Suppose you toss a coin and then roll a six-sided die. List the possible outcomes.



The Fundamental Counting Principle

If one event can occur m ways and a second event can occur n ways, the number of ways they can occur in a sequence is equal to

$$m \cdot n$$

Example: The access code for a security system is 4 digits. How many possible codes are there if:

Each digit can only be used once

$$10 \cdot 9 \cdot 8 \cdot 7 = 5040$$

There are no restrictions

$$10 \cdot 10 \cdot 10 \cdot 10 = 10,000$$

The first digit cannot be a 0 or a 1

$$8 \cdot 10 \cdot 10 \cdot 10 = 8000$$

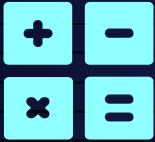
Example

You are buying a new car and you need to choose a manufacture, size, and color out of the following options:

- **Manufacture:** Ford, GM, Honda
- **Car Size:** Compact, Midsize
- **Color:** White, Red, Black, Green


$$3 \cdot 2 \cdot 4 = 24$$

Three types of probability



Classical Probability

Used when each outcome in a sample space is equally likely to occur



Empirical Probability

Based on observations from probability experiments.



Subjective Probability

Based on intuition, educated guesses, or estimates

Classical Probability

Note: this is also called theoretical probability

$$P(E) = \frac{\text{\# of outcomes in event } E \text{ (desired outcomes)}}{\text{\# of outcomes in sample space (possible outcomes)}}$$

Example: You roll a six-sided die. Find the probability of the following events:

- Event A: You roll a 3
- Event B: You roll a 7
- Event C: You roll a number less than 5

$$P(A) = \frac{1}{6}$$

$$P(B) = 0$$

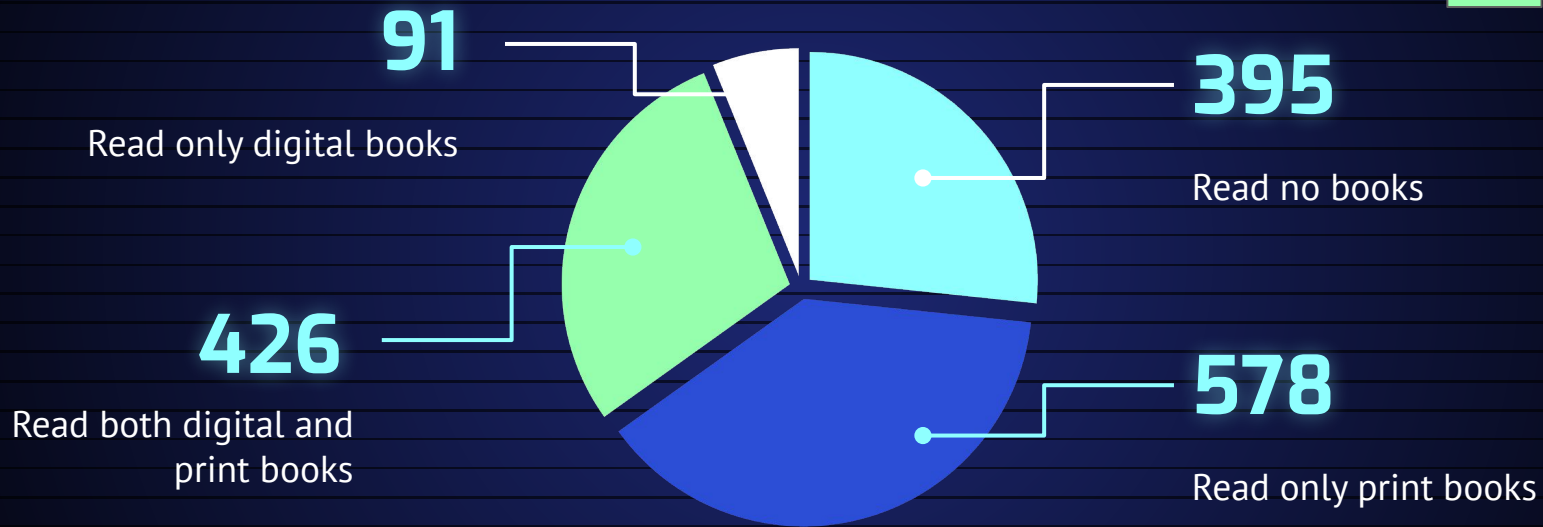
$$P(C) = \frac{4}{6} = \frac{2}{3}$$

Empirical Probability

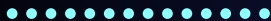
Note: this is also called statistical probability

$$P(E) = \frac{\text{frequency of event } E}{\text{total frequency}} = \frac{f}{n}$$

Example: A company surveyed people about their reading habits last year



The probability that the next adult questioned read only print books is: $\frac{578}{1490} \approx 0.388$ (38.8%)



Example: A company asks the ages of people who use a certain social network. Find the probability that the next user is between 23-35 years old.

<u>Ages</u>	<u>f</u>
18-22	156
23-35	312
36-49	254
50-64	195
65+	58

$$\frac{312}{975} = 0.32 \text{ (i.e. 32\%)}$$

Range of Probability Rule

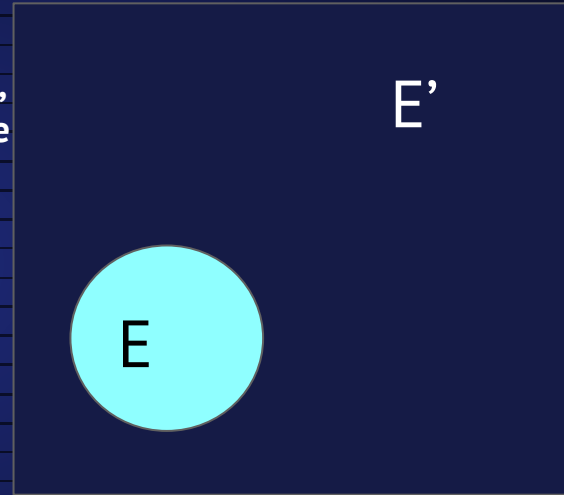
$$0 \leq P(E) \leq 1$$



Definition:

The complement of an event E , denoted E' (e-prime), is the set of the outcomes in a sample space that are not included in E

$$P(E) + P(E') = 1$$



Example: A company asks the ages of people who use a certain social network. Find the probability that the next user is NOT between 23-35 years old.

<u>Ages</u>	<u>f</u>
18-22	156
23-35	312
36-49	254
50-64	195
65+	58

$$1 - \frac{312}{975} = \frac{663}{975} = 0.68$$