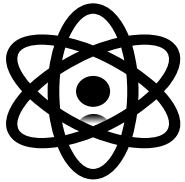
Chapter 3
Probability

ACSTA101 - Professor MG

3.1
Basic Concepts of Probability and Counting

Definitions

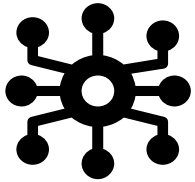


Experiment

A probability experiment is an action or trial through which specific results are obtained

Outcome

An outcome is a result of a probability experiment



Sample Space

The set of all possible outcomes is called the sample space

Event

An event is a subset of the sample space that consists of one or more outcomes.

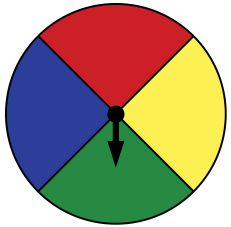
Example

A survey asked people for their blood types. Determine how many outcomes there are and identify the sample space.

There are 4 main types (A, B, AB, and O) that can be positive or negative. This gives 8 outcomes.

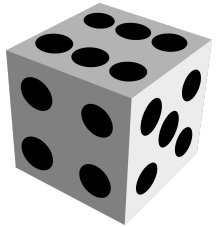
Sample space = {O+, O-, A+, A-, B+, B-, AB+, AB-}

More Examples



Probability Experiment: Spinning the spinner

4 possible outcomes.
Sample space = {red, yellow, blue, green}

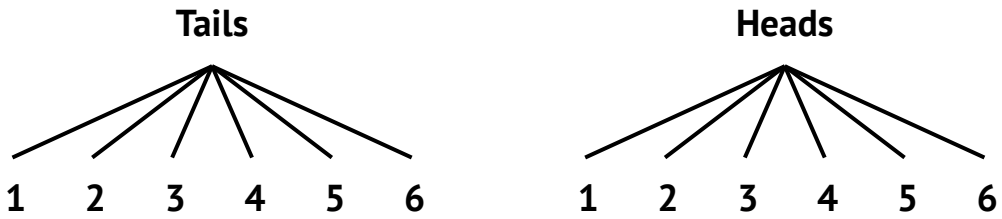


Probability experiment: Rolling a 6-sided die

6 possible outcomes.
Sample space = {1, 2, 3, 4, 5, 6}

More Complex Example

Suppose you toss a coin and then roll a six-sided die. List the possible outcomes.



The Fundamental Counting Principle

If one event can occur *m* ways and a second event can occur *n* ways, the number of ways they can occur in a sequence is equal to

$$m \cdot n$$

Example: The access code for a security system is 4 digits. How many possible codes are there if:

Each digit can only be used once

$10 \cdot 9 \cdot 8 \cdot 7 = 5040$

There are no restrictions

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$

The first digit cannot be a 0 or a 1

$8 \cdot 10 \cdot 10 \cdot 10 = 8000$

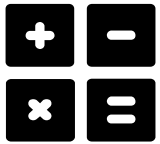
Example

You are buying a new car and you need to choose a manufacture, size, and color out of the following options:

- **Manufacture:** Ford, GM, Honda
- **Car Size**: Compact, Midsize
- **Color**: White, Red, Black, Green

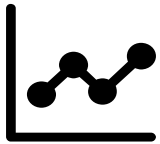
$$3 \cdot 2 \cdot 4 = 24$$

Three types of probability



Classical Probability

Used when each outcome in a sample space is equally likely to occur



Empirical Probability

Based on observations from probability experiments.

Subjective Probability

Based on intuition, educated guesses, or estimates

Classical Probability

Note: this is also called theoretical probability

$$P(E) = \frac{\#\text{ of outcomes in event E (desired outcomes)}}{\#\text{ of outcomes in sample space (possible outcomes)}}$$

Example: You roll a six-sided die. Find the probability of the following events:

- **Event A: You roll a 3**
- **Event B: You roll a 7**
- **Event C: You roll a number less than 5**

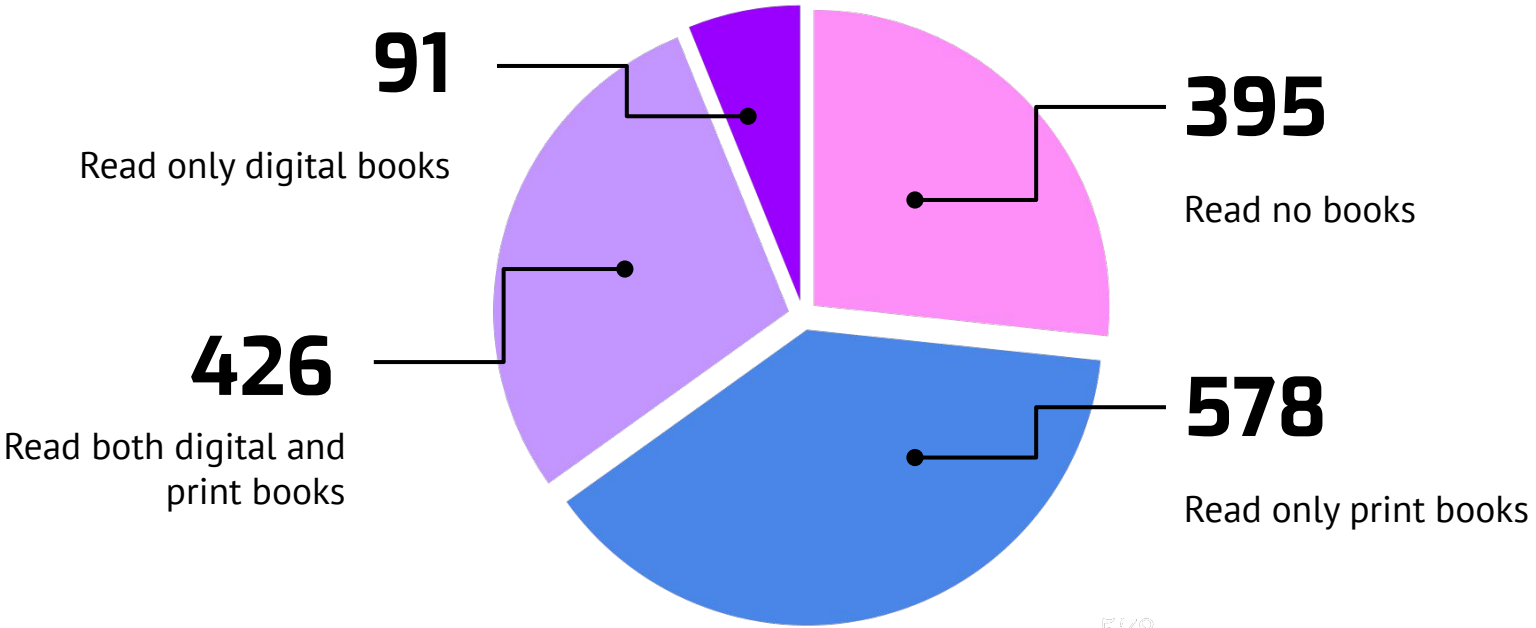
$$P(A) = \frac{1}{6} \qquad P(B) = 0 \qquad P(C) = \frac{4}{6} = \frac{2}{3}$$

Empirical Probability

Note: this is also called statistical probability

$$P(E) = \frac{\text{frequency of event E}}{\text{total frequency}} = \frac{f}{n}$$

Example: A company surveyed people about their reading habits last year



$$\frac{578}{1490} \approx 0.388\,(38.8\%)$$

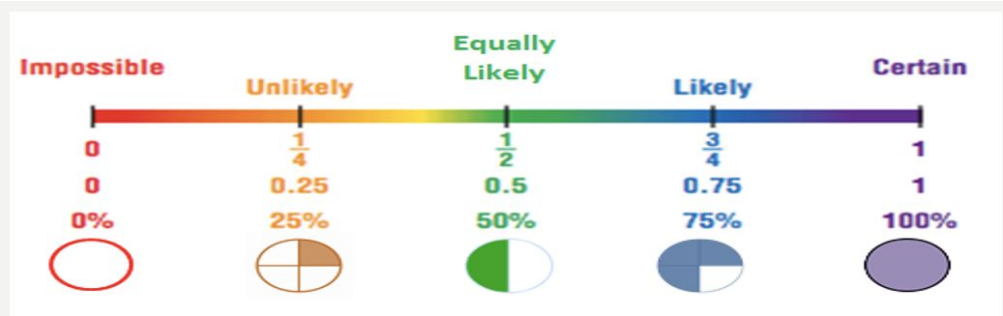
Example: A company asks the ages of people who use a certain social network. Find the probability that the next user is between 23-35 years old.

Ages	*f*
18-22	156
23-35	312
36-49	254
50-64	195
65+	58

$$\frac{312}{975} = 0.32 \,(\text{i.e. } 32\%)$$

Range of Probability Rule

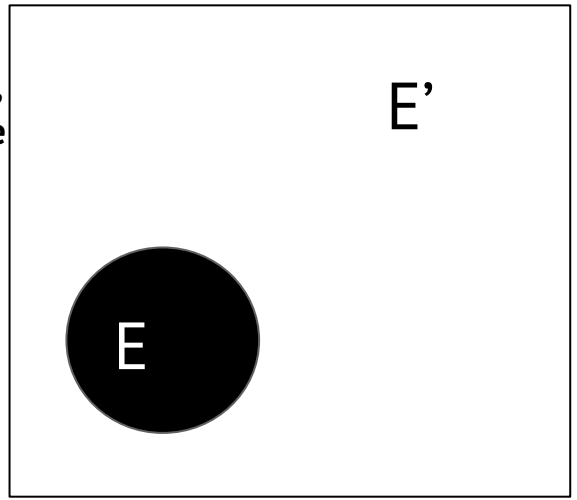
$$0 \leq P(E) \leq 1$$



Definition:

The <u>complement</u> of an event E, denoted E' (e-prime), is the set of the outcomes in a sample space that are not included in E

$$P(E) + P(E') = 1$$



Example: A company asks the ages of people who use a certain social network. Find the probability that the next user is NOT between 23-35 years old.

Ages	*f*
18-22	156
23-35	312
36-49	254
50-64	195
65+	58

$$1 - \frac{312}{975} = \frac{663}{975} = 0.68$$