3.2
Conditional Probability and the Multiplication Rule

Conditional Probability

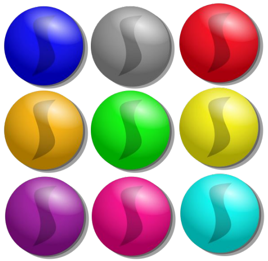
The probability of an event occurring, given that another event has already occurred

P(B|A) means "the probability of B, given A"

Some Examples

You choose two cards from a deck.

What is the probability that the second card you chose is a queen given that the first was a king (and you didn't put the king back in)?



You have 5 marbles, 2 are blue and 3 are red

If A is the event that you select a red marble and B is the event that you select a blue marble, find P(A|B) and P(B|A).

Example

You are studying the effect of a certain gene on an individual's IQ. You collect the data summarized on the right.

1. Find the probability that a person has a high IQ, given that they have the gene.
2. Find the probability that a person does not have the gene.
3. Find the probability that a person does not have the gene, given that they have a normal IQ.

	Has Gene	Doesn't have gene	Total
High IQ	33	19	52
Normal IQ	39	11	50
Total	72	30	102

Independent Events

Two events are independent when the occurrence of one event does not affect the probability of the occurrence of the other.

In other words, $P(B|A)=P(B)$ or $P(A|B)=P(A)$

For example: Flipping a coin two times. The second coin flip is independent from the first.

Example: Dependent VS. Independent

Selecting a king from a deck and then selecting a queen (without replacement)

Dependent

Independent

Tossing a coin and getting heads then rolling a six-sided die and getting 6

The Multiplication Rule

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

For independent events,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Examples

You select two cards (without replacement). Find the probability that you pick a king and a queen.

$$\frac{4}{52} \cdot \frac{4}{51} \approx 0.006$$

You flip a coin and roll a 6-sided die. Find the probability that you get a head and a 6.

$$\frac{1}{2} \cdot \frac{1}{6} \approx 0.083$$

Example: The probability of a reconstructive ACL surgery being successful is 95%. Find the following:

a) **The probability that 3 surgeries are successful**
b) **The probability that none of them are successful**
c) **The probability that at least one of them is successful**

a) $(0.95)(0.95)(0.95) \approx 0.857$

b) $(0.05)(0.05)(0.05) \approx 0.0001$

c) $1-0.0001 = 0.9999$