CHAPTER 4

DISCRETE PROBABILITY DISTRIBUTIONS

ACSTA101 - Professor MG

4.1 PROBABILITY DISTRIBUTIONS

THERE ARE TWO TYPES OF RANDOM VARIABLES

DISCRETE

Possible outcomes are finite or countable. Can only take on very specific values.

VS

CONTINUOUS

Has an uncountable number of possible outcomes. Can take on any value in an interval.

EXAMPLES

DISCRETE

The number of calls a salesperson makes in a day

DISCRETE

The number of Arrupe students who are pre-majoring in business

DISCRETE

The value you get when rolling a 6-sided die

CONTINUOUS

The amount of time spent on your phone in a day

CONTINUOUS

The volume of gas in a 21-gallon tank

CONTINUOUS

The height of an individual person

Definition: A discrete probability distribution lists each possible value together with its probability. It must satisfy the following:

1. $0 \le P(X) \le 1$
2. $\sum P(X) = 1$

CONSTRUCTING A DISCRETE PROBABILITY DISTRIBUTION

Let x be a discrete random variable with possible outcomes
$x_1, x_2, \ldots, x_n$

- Make a frequency distribution for the possible outcomes.
- Find the sum of the frequencies.
- Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
- You can also modify the stroke, its color, thickness or type
- Check that each probability is between 0 and 1 and that the sum is 1.

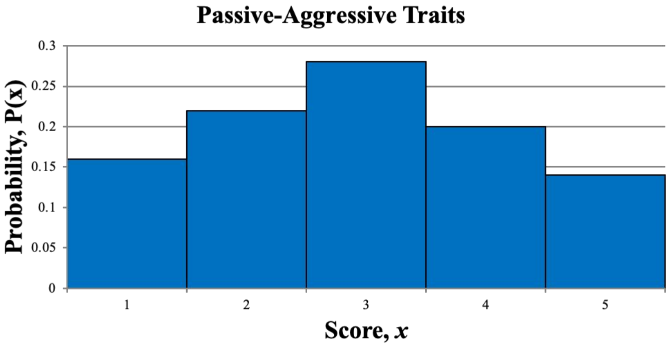
EXAMPLE

Score (x)	Freq (f)
1	24
2	33
3	42
4	30
5	21

An industrial psychologist administered a personality inventory test for passive aggressive traits to 150 employees. Individuals were given a score from 1 to 5, where 1 was extremely passive and 5 was extremely aggressive. A score of 3 indicated neither trait. Construct a probability distribution for the random variable x. Then graph the distribution using a histogram.

EXAMPLE CONTINUED

x	1	2	3	4	5
P(x)	0.16	0.22	0.28	0.20	0.14



Examples of interpreting this data:

- **The probability that a person scored 4 is 20%**
- **The probability that a person scored at least 3 is 62%**

MEAN

$$\mu = \sum xP(x)$$

VARIANCE

$$\sigma^2 = \sum (x-\mu)^2 P(x)$$

STANDARD DEVIATION

$$\sigma = \sqrt{\sum (x-\mu)^2 P(x)}$$

EXAMPLE CONT.

$$\mu = 1(0.16) + 2(0.22) + \ldots + 5(0.14) \approx 2.9$$

$$\begin{aligned}\sigma^2 &= (1-2.9)^2(0.16) + \ldots + (5-2.9)^2(0.14) \\ &\approx 1.6164\end{aligned}$$

$$\sigma = \sqrt{1.6164} \approx 1.3$$

EXPECTED VALUE

The expected value of a variable is equal to the mean. It is what you would expect the variable to tend towards in the long run

EXAMPLE

At a raffle, 1500 tickets are sold at $2 each for four prizes of $500, $250, $150, and $75. You buy one ticket. What is the expected value of your gain?

Gain	$498	$248	$148	$73	$-2
P(x)	1/1500	1/1500	1/1500	1/1500	1496/1500

$$
\begin{aligned}
E(x) &= 498\left(\frac{1}{1500}\right)+\ldots+(-2)\left(\frac{1496}{1500}\right) \\
&= \$-1.35
\end{aligned}
$$