4.2

BINOMIAL

DISTRIBUTIONS

DEFINITION: BINOMIAL EXPERIMENTS SATISFY:

01 REPETITION

The experiment is repeated a fixed number of times, where each trial is <u>independent</u>

02 2 OUTCOMES

There are only two possible outcomes of interest: success and failure

03 EQUAL PROBABILITY
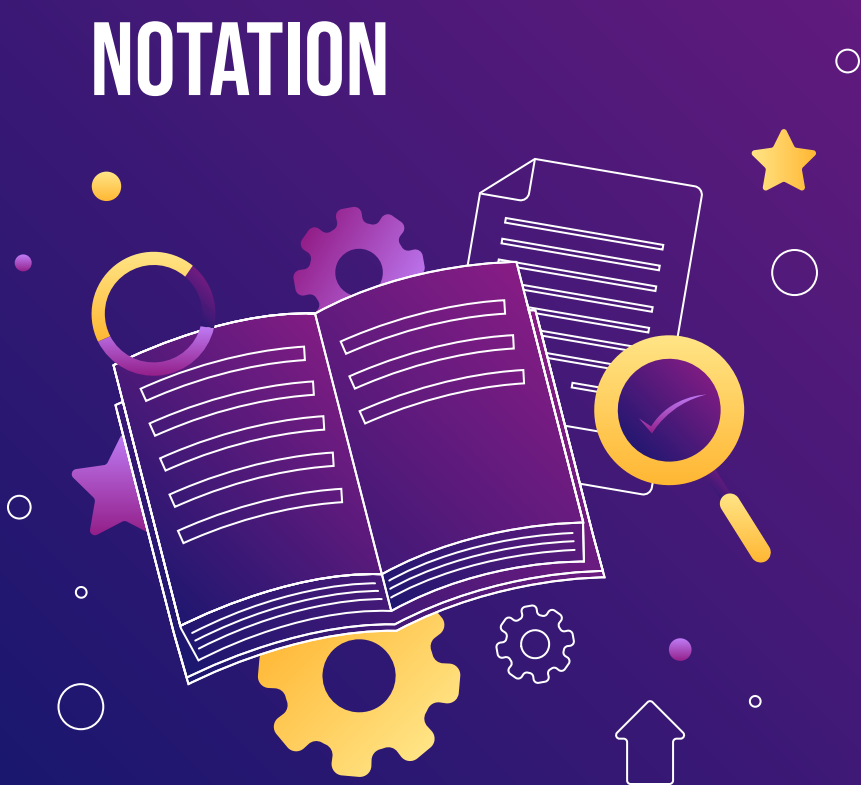
The probability of success is the same for each trial

04 X=# OF SUCCESSES

The random variable, x, counts the number of successful trials

NOTATION

The number of trials — n

The probability of success in a single trial — p

The probability of failure in a single trial — q

The number of successes after n trials — x

EXAMPLES

You pick 5 cards from a deck, replacing them each time before picking again. You record whether you got a club.

You conduct a procedure on 8 patients and record whether or not it was successful.

$$P(x) = {}_nC_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

Binomial Probability Formula

EXAMPLE 1

A surgery has a 90% chance of success and is performed on 3 patients. Find the probability that there are exactly 2 successes.

$$P(2) = \frac{3!}{(3-2)!2!}\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^1$$

$$= 2\left(\frac{81}{100}\right)\left(\frac{1}{10}\right)$$
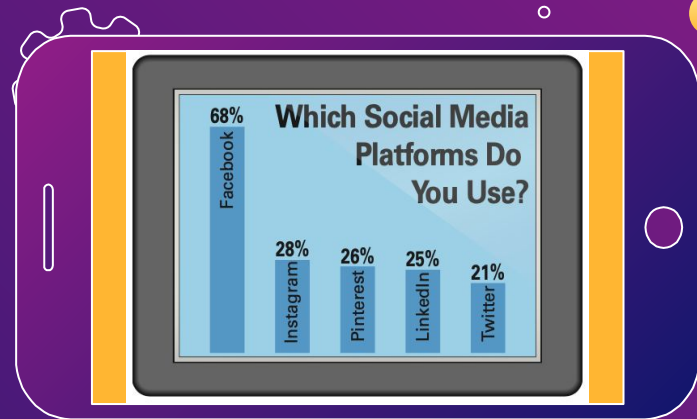
$$\approx 0.243$$

EXAMPLE 2

In a survey, US adults were asked to identify which social media platforms they use. Their responses are recorded to the right.

If 6 adults are randomly selected and asked if they use Facebook, construct a binomial distribution for the number of adults who said yes.

Which Social Media Platforms Do You Use?

68% Facebook
28% Instagram
26% Pinterest
25% LinkedIn
21% Twitter

MENU   ANALYSIS   CONTACT

EXAMPLE 2 CONTINUED

$$P(0) = {}_6C_0(0.68)^0(0.32)^6 \approx 0.001$$

$$P(1) = {}_6C_1(0.68)^1(0.32)^6 \approx 0.014$$

$$\vdots$$

$$P(6) = {}_6C_6(0.68)^6(0.32)^0 \approx 0.099$$

x	P(x)
1	0.001
2	0.014
3	0.073
4	0.206
5	0.279
6	0.099

EXAMPLE 3

A survey found that 17% of US adults say Google News is a major news source for them. You randomly select 4 adults. Find the following probabilities:

a) $P(2)$

b) $P(x \geq 2)$

c) $P(x < 2)$

Google News

0.119

$$P(2) = \,_4C_2(0.17)^2(0.83)^2 \approx 0.119$$

0.137

P(x≥2)=P(2)+P(3)+P(4)

0.863

P(x<2)=P(0)+P(1)

MEAN, VARIANCE, AND STANDARD DEVIATION

MEAN

$$\mu = np$$

VARIANCE

$$\sigma^2 = npq$$

STANDARD DEVIATION
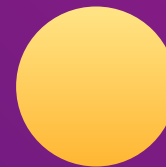
$$\sigma = \sqrt{npq}$$

EXAMPLE

About 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days in June.

MEAN

16.8

VARIANCE

7.4

STANDARD DEVIATION

2.7