

# **4.2 BINOMIAL DISTRIBUTIONS**

# DEFINITION: BINOMIAL EXPERIMENTS SATISFY:

01

## REPETITION

The experiment is repeated a fixed number of times, where each trial is independent

03

## EQUAL PROBABILITY

The probability of success is the same for each trial

02

## 2 OUTCOMES

There are only two possible outcomes of interest: success and failure

04

## $X = \#$ OF SUCCESSES

The random variable,  $x$ , counts the number of successful trials

# NOTATION

The number of trials  $n$

The probability of success  
in a single trial  $p$

The probability of failure  
in a single trial  $q$

The number of successes  
after  $n$  trials  $X$

# EXAMPLES

You pick 5 cards from a deck, replacing them each time before picking again. You record whether you got a club.

You conduct a procedure on 8 patients and record whether or not it was successful.

$$P(x) = {}_n\mathbf{C}_x p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

Binomial Probability Formula

# EXAMPLE 1

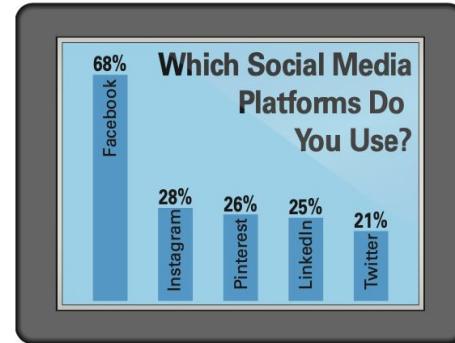
A surgery has a 90% chance of success and is performed on 3 patients. Find the probability that there are exactly 2 successes.

$$\begin{aligned}P(2) &= \frac{3!}{(3-2)!2!} \left(\frac{9}{10}\right)^2 \left(\frac{1}{10}\right)^1 \\&= 2 \left(\frac{81}{100}\right) \left(\frac{1}{10}\right) \\&\approx 0.243\end{aligned}$$

# EXAMPLE 2

In a survey, US adults were asked to identify which social media platforms they use. Their responses are recorded to the right.

If 6 adults are randomly selected and asked if they use Facebook, construct a binomial distribution for the number of adults who said yes.



## EXAMPLE 2 CONTINUED

$$P(0) = {}_6C_0(0.68)^0(0.32)^6 \approx 0.001$$

$$P(1) = {}_6C_1(0.68)^1(0.32)^5 \approx 0.014$$

⋮

$$P(6) = {}_6C_6(0.68)^6(0.32)^0 \approx 0.099$$

<u>x</u>	<u>P(x)</u>
<b>1</b>	<b>0.001</b>
<b>2</b>	<b>0.014</b>
<b>3</b>	<b>0.073</b>
<b>4</b>	<b>0.206</b>
<b>5</b>	<b>0.279</b>
<b>6</b>	<b>0.099</b>



# EXAMPLE 3

A survey found that 17% of US adults say Google News is a major news source for them. You randomly select 4 adults. Find the following probabilities:

- a)  $P(2)$
- b)  $P(x \geq 2)$
- c)  $P(x < 2)$

**0.119**

$$P(2) = {}_4C_2(0.17)^2(0.83)^2 \approx 0.119$$

**0.137**

$$P(x \geq 2) = P(2) + P(3) + P(4)$$

**0.863**

$$P(x < 2) = P(0) + P(1)$$

# MEAN, VARIANCE, AND STANDARD DEVIATION

**MEAN**

$$\mu = np$$

**VARIANCE**

$$\sigma^2 = npq$$

**STANDARD DEVIATION**

$$\sigma = \sqrt{npq}$$

# EXAMPLE

About 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days in June.

**MEAN**

16.8

**VARIANCE**

7.4

**STANDARD DEVIATION**

2.7