5.2 Normal Distributions: Finding Probabilities

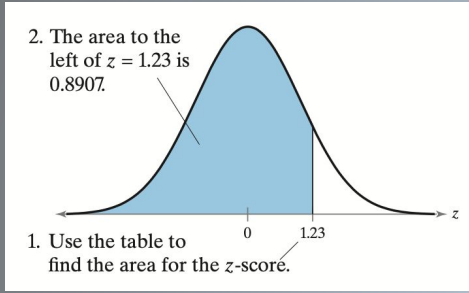
Recall

Finding the probability that *x* lies in a given interval of a normal distribution is the same thing as finding the area under the curve of that interval.

Steps:

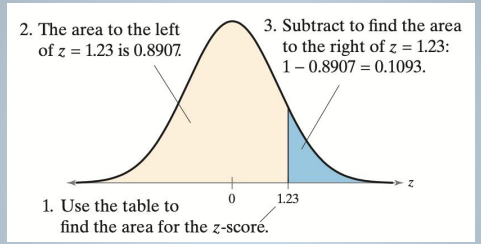
1. Convert the boundaries to a z-score
2. Determine which corresponding area under the curve you're looking for
3. Use a standard table or your calculator to find the area

Probability and Area



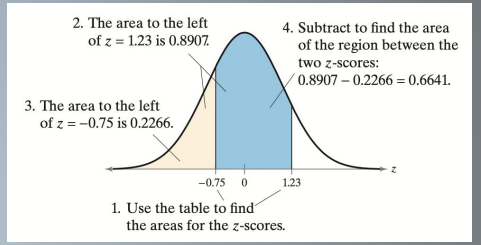
Area to the left

Same thing as
$P(x < z)$ or $P(x \leq z)$



Area to the right

Same thing as
$P(x > z)$ or $P(x \geq z)$



Area between two values

Same thing as
$P(z_1 < x < z_2)$ or
$P(z_1 \leq x \leq z_2)$

Example: A survey indicates that for each trip to the supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. The length of time spent in the store is normally distributed and is represented by the variable *x*. A shopper enters the store. Find:

a) The probability that the shopper spends between 22 and 52 min

b) The probability that the shopper spends more than 37 min

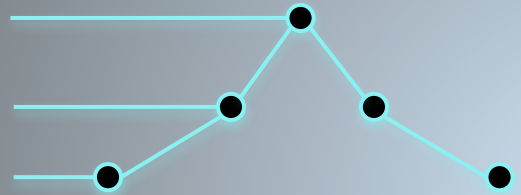
Example: A survey indicates that people use their cellular phones an average of 1.5 years before buying a new one. The standard deviation is 0.25 year. A cellular phone user is selected at random. Find the probability that the user will use their current phone for less than 1 year before buying a new one. Assume that the variable x is normally distributed.

Normal Distributions: Finding Values

5.3

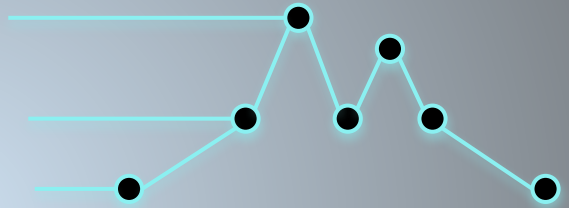
In the previous sections, you were given a value and asked to find an area/probability. But what if you had to do the opposite?

Steps to find a value given a probability:



Find the corresponding z-score

You can do this by looking at the closest value on a standard table and finding the z-score associated to that entry, or using technology



Convert the z-score to a value

Equation for this:

$x = \mu + z\sigma$

Practicing Step 1:

Find the z-score that corresponds to a cumulative area of 0.3632.

Find the z-score that has 10.75% of the distribution's area to its right.

Find the z-scores for which 68% of the distribution's area lies between $-z$ and z

Practicing Step 2:

A veterinarian records the weights of cats treated at a clinic. The weights are normally distributed, with a mean of 9 pounds and a standard deviation of 2 pounds. Find the weight x corresponding to each z-score below. Interpret the results.

$z = 1.96$, $z = -0.44$, and $z = 0$

Putting them together:

Example: Scores for the California Peace Officer Standards and Training test are normally distributed, with a mean of 50 and a standard deviation of 10. An agency will only hire applicants with scores in the top 10%. What is the lowest score you can earn and still be eligible to be hired by the agency?

Answer key:

Ex 1 (two curves)	Curve A has the greater mean, Curve B has the greater standard deviation
Ex 2 (cumul. areas)	0.8749, 0.4052, and 0.1611
Ex 3 (other areas)	0.1446 and 0.8275
Ex 4 (shop survey)	0.7333 and 0.6915
Ex 5 (cell phones)	0.0228
Ex 6 (step 1)	-0.35, 1.24, and 0.9945

Answer key continued:

Ex 7 (step 2) 12.92 lbs, 8.12 lbs, and 9 lbs

Ex 8 (peace officer) 62.8