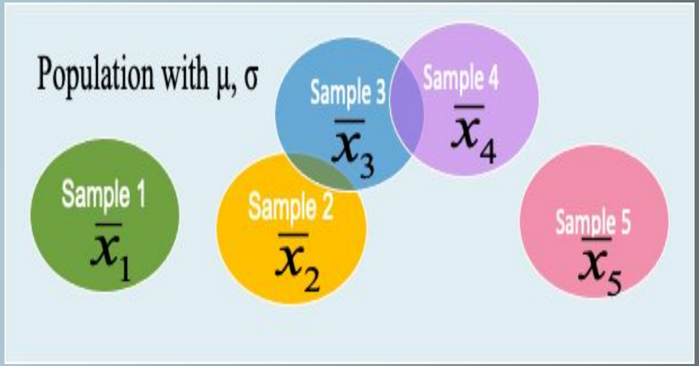
Sampling Distributions and the Central Limit Theorem

5.4

Definition



A sampling distribution is the probability distribution of a sample statistic. It is formed when random samples of size *n* are repeatedly taken from a population.

If the sample statistic is the sample mean, the distribution is called the <u>sampling distribution of sample means</u>

Example

The number of times four people go grocery shopping in a month is given by the population values {51, 3, 5, 76}. You randomly choose two of the four people, with replacement. List all possible samples of size $n = 2$ and calculate the mean of each. These means form the sampling distribution of the sample means. Find the mean, variance, and standard deviation of the sample means.

Example Cont.

Possible samples of size n=2 and their means

Sample	Sample mean
1,1	1
1,3	2
1,5	3
1,7	4
3,1	2
3,3	3
3,5	4
3,7	5

Sample	Sample mean
5,1	3
5,3	4
5,5	5
5,7	6
7,1	4
7,3	5
7,5	6
7,7	7

$$\mu_{\bar{x}} = 4$$

$$\sigma_{\bar{x}} \approx 1.58$$

$$(\sigma_{\bar{x}})^2 = 2.5$$

The Central Limit Theorem

1. If random samples of size $n \geq 30$ are drawn from any population, the sampling distribution of sample means is approximately a normal distribution.
2. If the population is already normally distributed, then the above is true for any sample size n

Properties of the Sampling Distribution of Sample Means

Mean

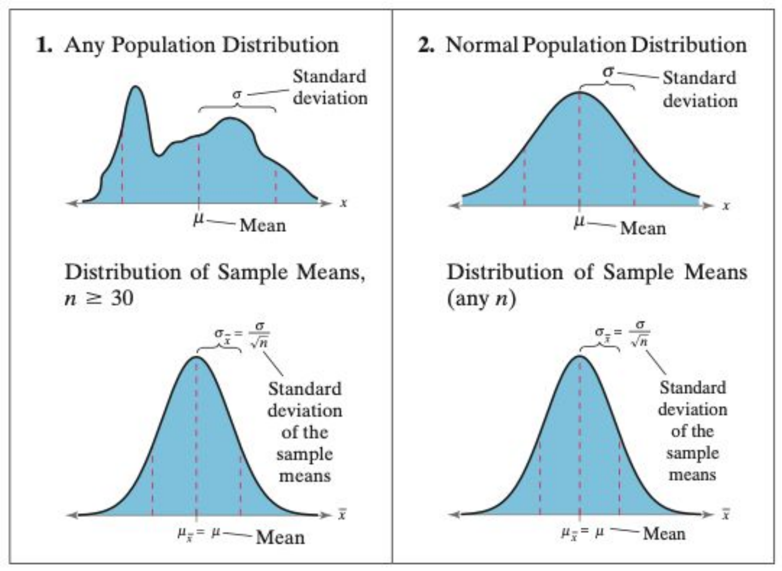
$$\mu_{\bar{x}} = \mu$$

Standard Deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Variation

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$$



Example

A study analyzed the sleep habits of college students. The study found that the mean sleep time was 6.8 hours, with a standard deviation of 1.4 hours. Random samples of 100 sleep times are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means.

$$\mu_{\bar{x}} = \mu = 6.8 \qquad \sigma_{\bar{x}} = \frac{1.4}{\sqrt{100}} = 0.14$$

Example

Assume the training heart rates of all 20-year-old athletes are normally distributed, with a mean of 135 beats per minute and a standard deviation of 18 beats per minute, as shown in the figure. Random samples of size 4 are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means. Then sketch a graph of the sampling distribution.

$$\mu_{\bar{x}} = \mu = 135$$

$$\sigma_{\bar{x}} = \frac{18}{\sqrt{4}} = 9$$

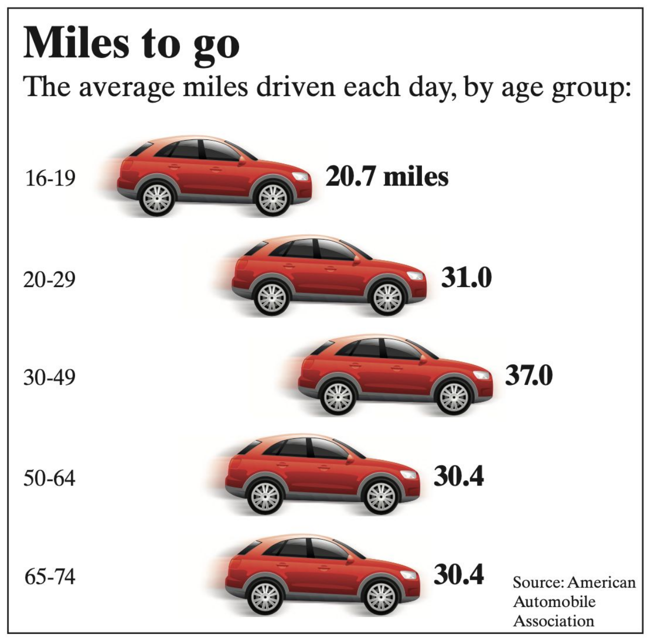
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}}$$

Probability and the Central Limit Theorem

Example

The figure at the right shows the mean distances traveled by drivers each day. You randomly select 50 drivers ages 16 to 19. What is the probability that the mean distance traveled each day is between 19.4 and 22.5 miles? Assume s = 6.5 miles.

Answer: 0.8957



Example

The mean room and board expense per year at four-year colleges is $10,453. You randomly select 9 four-year colleges. What is the probability that the mean room and board is less than $10,750? Assume that the room and board expenses are normally distributed with a standard deviation of $1650.

Answer: 0.7054