

6.2

Confidence Intervals for the Mean (σ known)

When the population standard deviation is unknown, we use what is called a t-distribution

If the distribution of a random variable x is approximately normal, then we look at the distribution of the variable t , defined as

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Properties of a t-distribution

Critical values of t are denoted t_c
The mean, median, and mode are equal to 0
t-distribution is bell-shaped and symmetric about the mean
The tails of a t-distribution are thicker than the standard normal distribution
The standard deviation of the t-distribution changes based on sample size but is always greater than 1

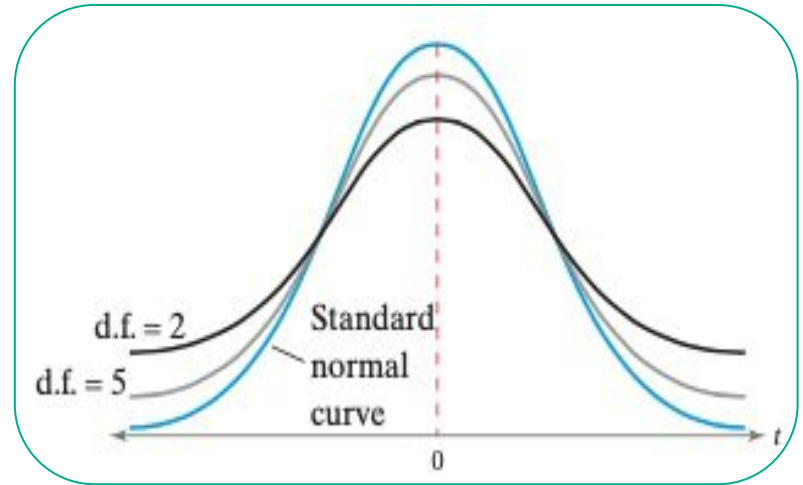
Definition

The degrees of freedom (d.f.) are the number of free choices left after a sample statistic is calculated.

For a t-distribution estimating population mean, the degrees of freedom is given by:

$$\text{d.f.} = n - 1$$

The larger the degrees of freedom, the closer the t-distribution is to a standard normal distribution



t-distribution Table

We use a table or a calculator to find the critical value (t_c) for t-distributions

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05
two-tails	1.00	0.50	0.40	0.30	0.20	0.10
df						
1	0.000	1.000	1.376	1.963	3.078	6.314
2	0.000	0.816	1.061	1.386	1.886	2.920
3	0.000	0.765	0.978	1.250	1.638	2.353
4	0.000	0.741	0.941	1.190	1.533	2.132
5	0.000	0.727	0.920	1.156	1.476	2.015
6	0.000	0.718	0.906	1.134	1.440	1.943
7	0.000	0.711	0.896	1.119	1.415	1.895
8	0.000	0.706	0.889	1.108	1.397	1.860
9	0.000	0.703	0.883	1.100	1.383	1.833
10	0.000	0.700	0.879	1.093	1.372	1.812

You can find the full table by searching “t-distribution table” online

Example

Find the critical value t_c for 95% confidence level when the sample size is 15

Answer: 2.145

New formula for error:

$$E = t_c \frac{s}{\sqrt{n}}$$

**This is only true if the sample is
randomly selected and the population is
approximately normal**

Steps for constructing a confidence interval when σ is known

01

Verify

Verify that the population standard deviation is NOT known, that the sample is random, and that the population is approximately normal

02

Calculate

Find the sample size, sample mean, and sample standard deviation

03

Calculate

Find the confidence level, degrees of freedom, and critical value. Use these to find the error.

04

Complete

Use the error to find the confidence interval

Example

You randomly select 16 coffee shops and measure the temperature of the coffee sold at each. The sample mean temperature is 162.0°F with a sample standard deviation of 10.0°F. Construct a 95% confidence interval for the population mean temperature of coffee sold. Assume the temperatures are approximately normally distributed.

$$\begin{aligned} E &= t_c \frac{s}{\sqrt{n}} \\ &= 2.131 \cdot \frac{10}{\sqrt{16}} \\ &\approx 5.3 \end{aligned} \quad (156.7, 167.3)$$