Confidence Intervals for Population Proportions

6.3

Recall

In section 4.2, we talked about binomial distributions.

p denoted the probability of success in a binomial experiment.

Definition: p is also called a population proportion.

This section will be about estimating p using interval estimates

Definition

A point estimate for population proportion p is given by $\hat{p}$ (p-hat), where p-hat is the sample proportion. i,e,

$$\hat{p} = \frac{x}{n} = \frac{\text{number of successes in the sample}}{\text{number in sample}}$$

Similarly, a point estimate for q (the population proportion of failures) is

$$\hat{q} = 1 - \hat{p}$$

Example

In a survey of 1550 U.S. adults, 1054 said that they use the social media website Facebook. Find a point estimate for the population proportion of U.S. adults who use Facebook.

Answer: 68%

New formula for error:

$$E = z_c \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

This is only true if the population is approximately normal, i.e. the following must be true:

$$n\hat{p} \geq 5 \text{ and } n\hat{q} \geq 5$$

Steps for constructing a confidence interval for p

01

Find

Read the problem to determine n (total amount of subjects) and x (total amount of successes)

02

Calculate

Use n and x to calculate p-hat, then verify that $n\hat{p} \geq 5 \text{ and } n\hat{q} \geq 5$

03

Calculate

Use the confidence level to find critical value z_c, then calculate the margin of error E

04

Complete

Use the error to find the confidence interval

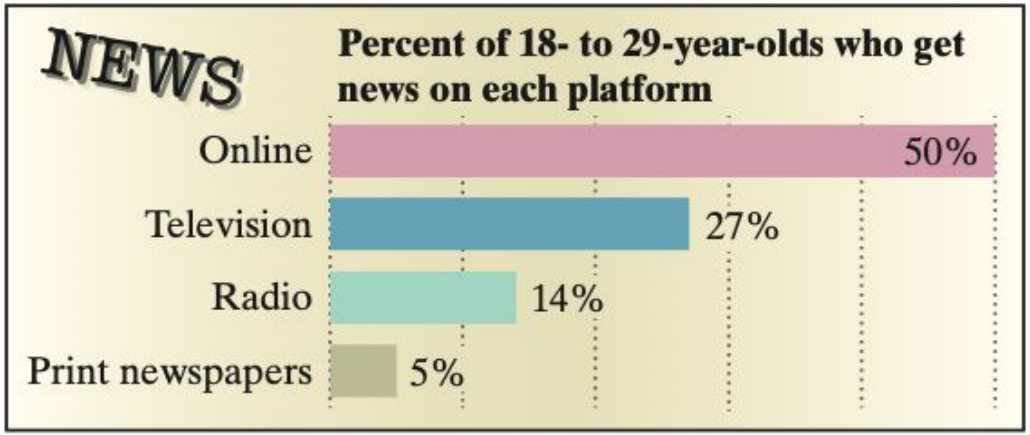
Example

Use the previous Facebook data to construct a 95% confidence interval for the proportion of adults who use Facebook.

Answer: (0.657, 0.703)

Example

The figure below is from a survey of 800 U.S. adults ages 18 to 29. Construct a 99% confidence interval for the population proportion of 18- to 29-year-olds who get their news on television.



p-hat = 0.27, q-hat = 0.73

$z_c = 2.575$

E ≈ 0.040

Confidence interval: (0.230, 0.310)

Given a c confidence level and a margin of error E, the minimum sample size, n, needed to estimate the population proportion is given by

$$n = \hat{p}\,\hat{q}\left(\frac{z_c}{E}\right)^2$$

Round up to the nearest whole number, if necessary

If you do not have an estimate for p-hat and q-hat, use 0.5 for both

Example

You are running a political campaign and wish to estimate, with 95% confidence, the population proportion of registered voters who will vote for your candidate. Your estimate must be accurate within 3% of the population proportion. Find the minimum sample size needed when (1) no preliminary estimate is available and (2) a preliminary estimate gives p-hat = 0.31. Compare your results.

For (1), p-hat and q-hat are unknown.

$$n = (0.5)(0.5)\left(\frac{1.96}{0.03}\right)^2 \approx 1067.11$$

Round up to 1068 people.

For (2), p-hat = 0.31

$$n = (0.31)(0.69)\left(\frac{1.96}{0.03}\right)^2 \approx 913.02$$

Round up to 914 people.