Chapter 7: Hypothesis Testing with One Sample

ACSTA101 - Professor MG

01 Introduction to Hypothesis Testing

Definition: Hypothesis Test

A hypothesis test is a process that uses sample statistics to test a claim about the value of a population parameter.

For example, If a manufacturer advertises that its new hybrid car has a mean gas mileage of 50 miles per gallon, you might want to collect a sample of cars to test this claim.

Definitions

A statement about a population parameter is called a statistical hypothesis. To test a parameter, we state two hypothesis. If one is false, the other must be true.

A null hypothesis (H_0)

- A statistical hypothesis that contains a statement of equality
- It may contain the following symbols: ≤, =, or ≥

An alternative hypothesis (H_a)

- The complement of the null hypothesis
- It may contain the following symbols: > , ≠, or <

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.* *. . . not shorter than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.* *. . . shorter than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.* *. . . not longer than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.* *. . . longer than k.*
. . . equal to k. *. . . k.* *. . . exactly k.* *. . . the same as k.* *. . . not changed from k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.* *. . . different from k.* *. . . changed from k.*

Examples: State the null and alternative hypotheses for the following. Then determine which hypothesis is the claim.

A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

$H_0 : p = 0.61$
$H_a : p \neq 0.61$

A company advertises that the mean life of its furnaces is more than 18 years.

$H_0 : \mu \geq 15$
$H_a : \mu < 15$

A car dealership announces that the mean time for an oil change is less than 15 minutes.

$H_0 : \mu \leq 18$
$H_a : \mu > 18$

Testing a Hypothesis

To test the null and alternative hypotheses, we always start by assuming H_0 is true. We then use a random sample to either:

1. Reject the null hypothesis, or
2. Fail to reject the null hypothesis

Because we're only looking at a sample, it is always possible that we will make the wrong decision, we call these mistakes Type I or Type II errors.

Possible Errors

Type I Error

- A Type I error occurs if the null hypothesis is rejected when it is actually true.

Type II Error

- A Type II error occurs if the null hypothesis is not rejected when it is actually false.

	Truth of H_0	
Decision	**H_0 is true.**	**H_0 is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Conducting Hypothesis Tests

In a hypothesis test, you'll need a level of significance (denoted α). It signifies the maximum probability you want to allow for making a Type I error.

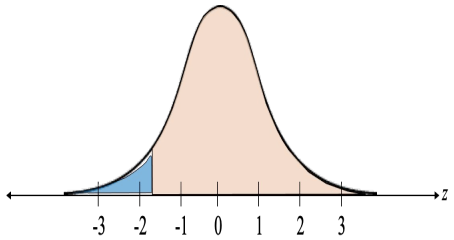
Typical α values are 0.10, 0.05, and 0.01.

Once you have the hypotheses and significance level, collect the necessary test statistic from a random sample and convert it to a standardized value

Which standardized value?

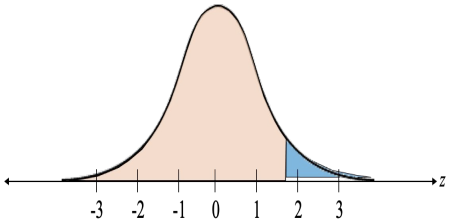
- If you're testing a claim about population mean (μ), you will look at $\bar{x}$ (sample mean) and convert it to either a critical value z (if σ is known) or t (if σ is unknown)
- If you're testing a claim about population proportion (p), you will look at $\hat{p}$ and convert to a critical value z.

Types of Hypothesis Tests



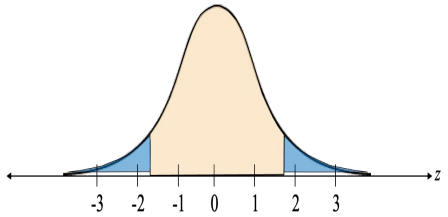
Left-Tailed

Use when the alternative hypothesis uses a < symbol.



Right-Tailed

Use when the alternative hypothesis uses a > symbol.



Two-Tailed

Use when the alternative hypothesis uses a ≠ symbol.

Examples: Determine which type of hypothesis test to use.

A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

$H_0 : p = 0.61$
$H_a : p \neq 0.61$

Two-tailed test

A company advertises that the mean life of its furnaces is more than 18 years.

$H_0 : \mu \geq 15$
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Left-tailed test

A car dealership announces that the mean time for an oil change is less than 15 minutes.

$H_0 : \mu \leq 18$
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Right-tailed test