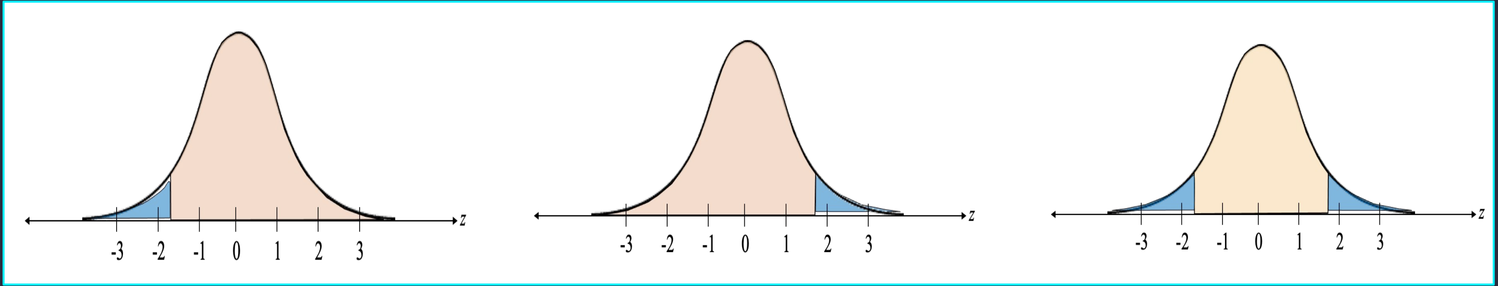
7.2 Hypothesis Testing for the mean

(σ known)

Definition: P-Value



Left-Tailed

The P value in a left-tailed test is the area to the left of the standardized test statistic

Right-Tailed

The P value in a right-tailed test is the area to the right of the standardized test statistic

Two-Tailed

The P value in a two-tailed test is the area in both tails i.e.
2*(area in one tail)

Recall for sample mean:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

Interpreting a decision

	Claim is H_0	Claim is H_a
Reject H_0	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to reject H_0	There is not enough evidence to reject the claim	There is not enough evidence to support the claim

Method 1: P-Value

Steps	
1	Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α).
4	Find the standardized test statistic (z).
5	Find the P value.
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at α = 0.01? Use a P-value.

H_0: $\mu \geq 13$

H_a: $\mu < 13$ (claim)

$\alpha = 0.01$

$$z = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

P=0.0014 < α so we reject H_0 (i.e. there is enough evidence to support the claim)

Example:

According to a study of U.S. homes that use heating equipment, the mean indoor temperature at night during winter is 68.3°F. You think this information is incorrect. You randomly select 25 U.S. homes that use heating equipment in the winter and find that the mean indoor temperature at night is 67.2°F. From past studies, the population standard deviation is known to be 3.5°F and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P-value

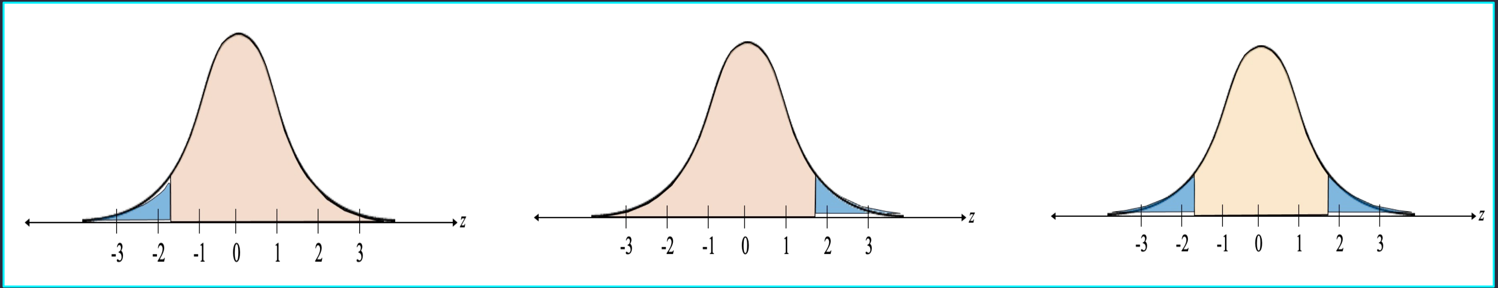
Answer: There is not enough evidence to support your claim

Definitions:

A rejection (or critical) region is the range of values for with H_0 is not probable. You reject H_0 if your statistic falls in the rejection region.

The critical value that separates the rejection region from the rest is denoted z_0

Rejection Regions: The area of the region should be equal to α



Left-Tailed

The rejection region is the area to the left of z_0

Right-Tailed

The rejection region is the area to the right of z_0

Two-Tailed

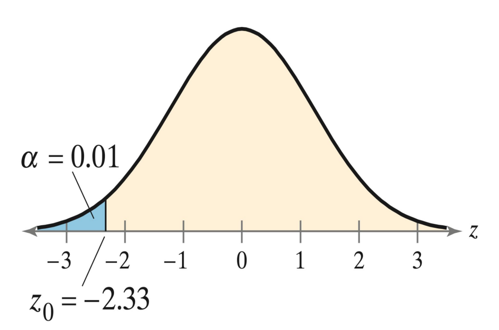
The rejection regions are the areas to the left of the first z_0 and the area to the right of the second

Example: Find the z_0 and rejection region for a left-tailed test with $\alpha = 0.01$

Alpha	**Tail**	***z***
0.10	Left Right Two	-1.28 1.28 ± 1.645
0.05	Left Right Two	-1.645 1.645 ± 1.96
0.01	Left Right Two	-2.33 2.33 ± 2.575

So $z_0 = -2.33$

The table to the left shows common z_0 values



Method 2: Rejection Regions

Steps	
1	Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α).
4	Determine the critical value(s) and the rejection region(s).
5	Find the standardized test statistic (z).
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P-value.

H_0: $\mu \geq 13$

H_a: $\mu < 13$ (claim)

$\alpha = 0.01$

Using the table for z_0 values, we get z_0=-2.33 and the rejection region is any z-value less than this

$$z = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

Our $z<z_0$, so we reject H_0 (i.e. there is enough evidence to support the claim)