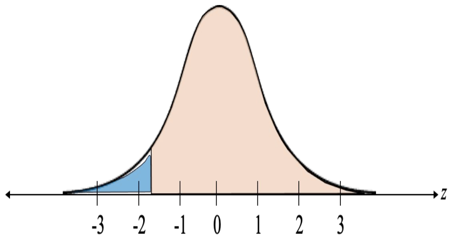
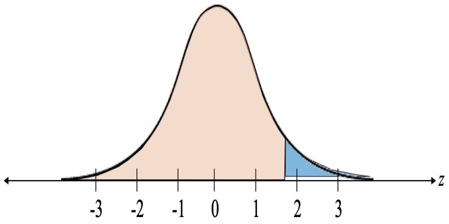
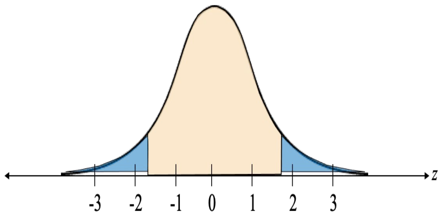
7.2 Hypothesis Testing for the mean

(σ known)

Definition: P-Value







Left-Tailed

The P value in a left-tailed test is the area to the left of the standardized test statistic

Right-Tailed

The P value in a right-tailed test is the area to the right of the standardized test statistic

Two-Tailed

The P value in a two-tailed test is the area in both tails i.e. 2*(area in one tail)

Recall for sample mean:

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Interpreting a decision

	Claim is H_0	Claim is H_a
Reject H_0	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to reject H_0	There is not enough evidence to reject the claim	There is not enough evidence to support the claim

Method 1: P-Value

Steps	
1	Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α).
4	Find the standardized test statistic (z).
5	Find the P value.
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P-value.

H_0: $\mu \geq 13$

H_a: $\mu < 13$ (claim)

$\alpha = 0.01$

$$z = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

P=0.0014 < α so we reject H_0 (i.e. there is enough evidence to support the claim)

Example:

According to a study of U.S. homes that use heating equipment, the mean indoor temperature at night during winter is 68.3ºF. You think this information is incorrect. You randomly select 25 U.S. homes that use heating equipment in the winter and find that the mean indoor temperature at night is 67.2ºF. From past studies, the population standard deviation is known to be 3.5ºF and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a P-value

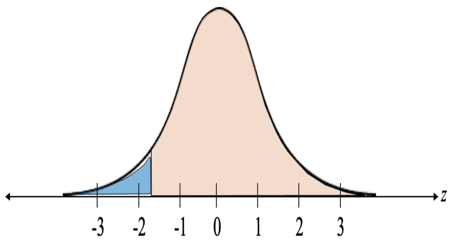
Answer: There is not enough evidence to support your claim

Definitions:

A rejection (or critical) region is the range of values for with H_0 is not probable. You reject H_0 if your statistic falls in the rejection region.

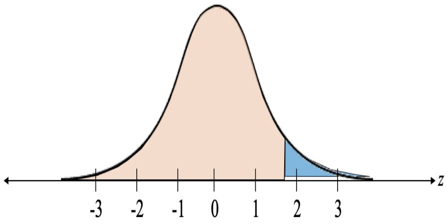
The critical value that separates the rejection region from the rest is denoted z_0

Rejection Regions: The area of the region should be equal to α



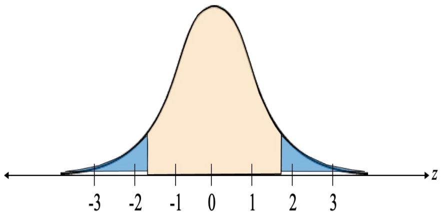
Left-Tailed

The rejection region is the area to the left of z_0



Right-Tailed

The rejection region is the area to the right of z_0



Two-Tailed

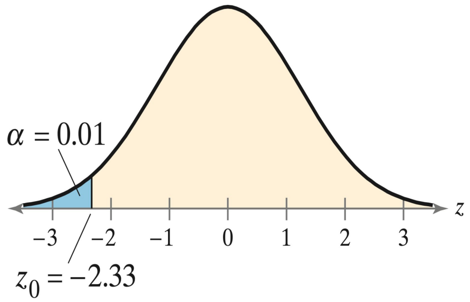
The rejection regions are the areas to the left of the first z_0 and the area to the right of the second

Example: Find the z_0 and rejection region for a left-tailed test with α=0.01

Alpha	Tail	z
0.10	Left	−1.28
	Right	1.28
	Two	±1.645
0.05	Left	−1.645
	Right	1.645
	Two	±1.96
0.01	Left	−2.33
	Right	2.33
	Two	±2.575

So z_0=-2.33

The table to the left shows common z_0 values



Method 2: Rejection Regions

Steps	
1	Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α).
4	Determine the critical value(s) and the rejection region(s).
5	Find the standardized test statistic (z).
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a P-value.

H_0: $\mu \geq 13$

H_a: $\mu < 13$ (claim)

$\alpha = 0.01$

Using the table for z_0 values, we get $z_0=-2.33$ and the rejection region is any z-value less than this

$$z = \frac{12.9 - 13}{0.19/\sqrt{32}} \approx -2.98$$

Our $z<z_0$, so we reject H_0 (i.e. there is enough evidence to support the claim)