7.3 Hypothesis Testing for the Mean

(σ unknown)

Finding Critical Value (t_0)

Recall: We often don't know the population standard deviation (σ). When this occurs, we use a t-test

To find the critical value for hypothesis testing (t_0):

1. Specify the level of significance (α)
2. Find the degrees of freedom (d.f. = n-1)
3. Use the t-table to find t_0

Examples

-1.725

t_0 for a left-tailed test with $\alpha = 0.05$ and $n = 21$

2.583

t_0 for a right-tailed test with $\alpha = 0.01$ and $n = 17$

±1.708

$-t_0$ and t_0 for a two-tailed test with $\alpha = 0.10$ and $n = 26$

Recall, if the sample is random and approximately normal:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Hypothesis Testing: Rejection Regions

Steps	
1	Verify that σ is not known, the sample is random, and either the population is normally distributed or $n \geq 30$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α) and find the degrees of freedom.
4	Determine the critical value(s) and the rejection region(s).
5	Find the standardized test statistic (t).
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

A used car dealer says that the mean price of used cars sold in the last 12 months is at least $21,000. You suspect this claim is incorrect and find that a random sample of 14 used cars sold in the last 12 months has a mean price of $19,189 and a standard deviation of $2950. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed.

$H_0: \mu \geq 21000$ (claim)

$H_a: \mu < 21000$

$\alpha = 0.05$

d.f. = 14-1 = 13

Using the table for t_0 values, we get t_0=-1.771 and the rejection region is any t-value less than this

$$t = \frac{19189 - 21000}{2950/\sqrt{14}} \approx -2.297$$

Our $t<t_0$, so we reject H_0 (i.e. there is enough evidence to reject the claim)

Example

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$?

H_0: $\mu = 6.8$ (claim)

H_a: $\mu \neq 6.8$

$\alpha = 0.05$

d.f. = 39-1 = 38

Using the table for t_0 values, we get $-t_0 = -2.024$, $t_0 = 2.024$, and the rejection region is any t-value less than -2.024 or greater than 2.024

$$t = \frac{6.7 - 6.8}{0.35/\sqrt{39}} \approx -1.784$$

Our t is not in our rejection region, so we fail to reject H_0 (i.e. there is not enough evidence to reject the claim)