Hypothesis Testing for Proportions

7.4

For population proportion,
if np≥5 and nq≥5, then:

$$z = \frac{\hat{p} - p}{\sqrt{pq/_n}}$$

Hypothesis Testing: Rejection Regions

Steps	
1	Verify that $np \geq 5$ and $nq \geq 5$.
2	State the claim mathematically and verbally. Identify the null and alternative hypotheses.
3	Specify the level of significance (α).
4	Determine the critical value(s) and the rejection region(s).
5	Find the standardized test statistic (z).
6	Make a decision to reject or fail to reject the null hypothesis.Interpret the decision in the context of the original claim.

Example

A researcher claims that less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember. In a random sample of 100 adults, 41% say they use passwords that are less secure because complicated ones are too hard to remember. At α = 0.01, is there enough evidence to support the researcher's claim?

np = 100(0.45) = 45 and nq = 100(0.55) = 55

H_0: $p \geq 0.45$

H_a: $p < 0.45$ (claim)

α = 0.01

Using the table for z_0 values, we get $z_0 = -2.33$ and the rejection region is any z-value less than this

$$z = \frac{0.41 - 0.45}{\sqrt{(0.45)(0.55)/100}}$$
$$\approx -0.80$$

Our $z > z_0$, so we fail to reject H_0 (i.e. there is not enough evidence to support the claim)

Example

A researcher claims that 51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses. In a random sample of 2202 adults, 1161 say antibiotics are effective against viruses. At α = 0.10, is there enough evidence to support the researcher's claim?

np = 2202(0.51) = 1123 and nq = 2202(0.49) = 1079

H_0: p = 0.51 (claim)

H_a: p ≠ 0.51

α = 0.10

$\hat{p}$ = 1161/2202 ≈ 0.527

Using the table for z_0 values, we get $-z_0$=-1.645, t_0 = 1.645, and the rejection region is any t-value less than -1.645 or greater than 1.645

$$z = \frac{0.527 - 0.51}{\sqrt{(0.51)(0.49)/2202}} \approx 1.60$$

Our z is not in the region of rejection, so we fail to reject H_0 (i.e. there is not enough evidence to reject the claim)