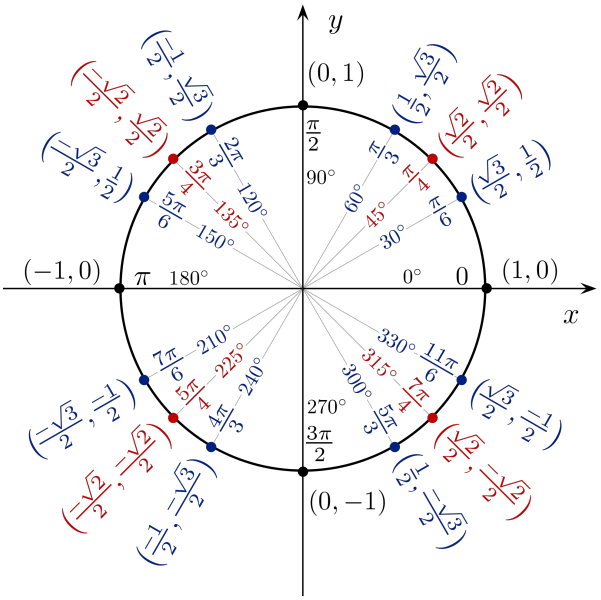
Sections 6.5 Lecture Notes

Recall: The Unit Circle

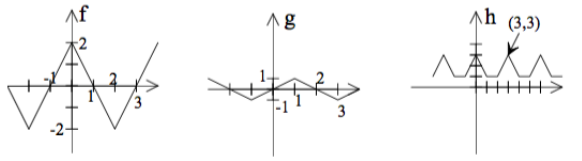


Exercise: Use the unit circle to graph $y = \sin x$, $y = \cos x$ and $y = \tan x$, where x is the angle measure in radians. Check your answer with a graphing utility.

Definitions:

- The *amplitude* of a function f is half the difference between the max and the min values of f.
- The *period* of a function is the smallest positive number where the function values start to repeat.

Example:



amplitude = 2	*amplitude* = 1	*amplitude* = 1
period = 4	*period* = 4	*period* = 3

Exercise: What are the amplitudes and periods for $\sin x$, $\cos x$, and $\tan x$?

Graphing Exercise: Use a graphing utility to graph the following functions:

- $y = 2\sin x$
- $y = 3\cos x$
- $y = 2\tan x$
- $y = 5\sin x$
- $y = 6\cos x$
- $y = 7\tan x$

Brainstorm: How does the number in front of the $\sin x$ and $\cos x$ affect the graph?

Graphing Exercise: Use a graphing utility to graph the following functions:

- $y = \sin 2x$
- $y = \cos \pi x$
- $y = \tan 2x$
- $y = \sin \frac{x}{4}$
- $y = \cos \frac{x}{2}$
- $y = \tan \frac{x}{2}$

Brainstorm: How does the number in front of the x affect the graph?

Brainstorm: Putting the previous two brainstorms together, what can we say about graphs of functions that looks like $f(x) = A \sin Bx$, $f(x) = A \cos Bx$, $f(x) = A \tan Bx$?

Recall: In your previous course(s), you learned about vertical and horizontal shifts (i.e. translations) of graphs as well as vertical and horizontal reflections. What do you remember?

Graphing Exercise: Use a graphing utility to graph the following functions:

- $y = \sin\left(x - \frac{\pi}{2}\right)$
- $y = -3\cos(\pi x)$
- $y = 4\tan(3x + 2)$
- $y = \sin(2x - \pi)]$
- $y = 2\cos\left[2\pi\left(x - \frac{1}{4}\right)\right]$
- $y = -\tan\left(3\left(x + \frac{\pi}{6}\right)\right)$

Brainstorm: Based on the above graphs, what strategies can we use to graph functions that look like the following?

- $f(x) = \pm A\sin(Bx \pm C) \pm D$
- $f(x) = \pm A\cos(Bx \pm C) \pm D$
- $f(x) = \pm A\tan(Bx \pm C) \pm D$

Definition: When graphing trigonometric functions, horizontal shifts are referred to as *phase shifts*.

Graphing Basic Trigonometric Functions: A Summary

Exercises: Graph the following functions over one or more periods. List the amplitudes, periods, phase shifts, and vertical shifts:

(1) $y = -\tan\left(\frac{\pi}{4}x + \frac{\pi}{8}\right)$ (2) $y = -2\sin\left(\frac{x}{4} - \frac{\pi}{2}\right) - 2$ (3) $y = \sin\left(3x + \frac{\pi}{2}\right) + 1$