Sections 7.3 Lecture Notes

Solving Trigonometric Equations: When solving trigonometric equations, we have to remember the fact that trigonometric functions are periodic.

Theorem/Reminder: For integers n:

$$\sin(\theta \pm n\pi) = \begin{cases} \sin\theta, & \text{if } n \text{ is even} \\ -\sin\theta, & \text{if } n \text{ is odd} \end{cases}$$

$$\cos(\theta \pm n\pi) = \begin{cases} \cos\theta, & \text{if } n \text{ is even} \\ -\cos\theta, & \text{if } n \text{ is odd} \end{cases}$$

$\tan(\theta \pm n\pi) = \tan\theta$, for all n

Example: Solve $\cos\theta = 1$.

Solution: Looking at the unit circle, we see that cosine is 1 at $\theta = 0$ and $\theta = 2\pi$. But, this is also true of any angle in that position, including angles like -2π, 4π, and 198π.

Brainstorm: If we're only looking for solutions between $[0, 2\pi]$, then we can look at the unit circle, but how do we account for the infinitely many solutions that exist outside of that?

Exercises: Find all solutions:

(1) $\sin\theta = \frac{1}{2}$

(2) $2\cot t + 1 = -1$

(3) $2\sin^2 t - 5\sin t + 2 = 0$

(4) $\cos^2 x + \cos x - 6 = 0$

For some trigonometric equations, you may have to use trigonometric identities to rewrite the problem before solving it.

Exercises:

(1) $2\tan\theta = \sec^2\theta$

(2) $\sin^2 x + \cos x + 1 = 0$

(3) $2\tan^2 x - 3\tan x \sec x - 2\sec^2 x = 0$