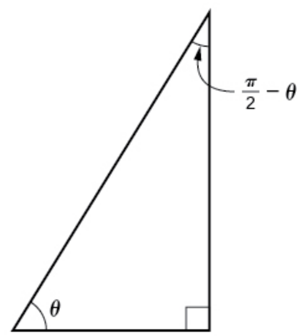
Sections 7.4 Lecture Notes

Co-function Identities:

- $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
- $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
- $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$
- $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$
- $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Brainstorm: Use the following picture of a triangle to understand why the above identities are true:



Sum and Difference Identities (Trigonometric Addition Formulas):

- $\sin(s + t) = \sin s \cos t + \cos s \sin t$
- $\sin(s - t) = \sin s \cos t - \cos s \sin t$
- $\cos(s + t) = \cos s \cos t - \sin s \sin t$
- $\cos(s - t) = \cos s \cos t + \sin s \sin t$
- $\tan(s + t) = \dfrac{\tan s + \tan t}{1 - \tan s \tan t}$
- $\tan(s - t) = \dfrac{\tan s - \tan t}{1 + \tan s \tan t}$

Brainstorm: How can we use plus/minus notation ($\pm$ and $\mp$) to turn the above 6 equations into 3?

Exercise: Use the sum and difference identities to prove at least one of the co-function identities.

Exercises (finding new trigonometric values): Use the sum and difference formulas to find the exact value of the following functions

(1) $\cos 15°$

(2) $\tan\left(\frac{\pi}{12}\right)$

Exercise (applying the identities):

(1) Let $\sin\alpha = \frac{4}{5}$ and $\cos\beta = \frac{3}{5}$. Find the following:

   (a) $\sin(\alpha + \beta)$

   (b) $\cos(\alpha + \beta)$

   (c) $\tan(\alpha + \beta)$

Exercise (going backward): Simplify the following by applying a sum/difference identity

(1) $\cos\frac{\pi}{5}\cos\frac{\pi}{30}+\sin\frac{\pi}{5}\sin\frac{\pi}{30}$

(2) $\sin(x+y)\cos y-\cos(x+y)\sin y$

(3) $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$

(4) $\dfrac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$

Exercise (trigonometric proofs): Verify the following

(1) $\sin(\theta + \pi) = -\sin\theta$

(2) $\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} = \cot\beta - \cot\alpha$

(3) $\sin(a+b) + \sin(a-b) = 2\sin a\cos b$

(4) $\cos(a-b) - \cos(a+b) = 2\sin a\sin b$