Sections 7.5 Lecture Notes

Double Angle Identities:

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  $= 2\cos^2\theta - 1$
  $= 1 - 2\sin^2\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Brainstorm: Use the sum/difference identities from last class to prove the above identities:

Half-Angle Identities:

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\theta}{1+\cos\theta}$

Brainstorm: Given information about θ, how can we determine the signs of $\sin\left(\frac{\theta}{2}\right)$ and $\cos\left(\frac{\theta}{2}\right)$?

Exercises:

(1) Given $\cos\theta = \frac{1}{3}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin 2\theta$, $\cos 2\theta$, $\sin\frac{\theta}{2}$, and $\cos\frac{\theta}{2}$

(2) Given $\cos\theta = -\frac{12}{13}$ and $\sin\theta > 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

(3) Find sine, cosine, and tangent for $\theta = \frac{\pi}{12}$.

More Exercises:

(1) Write $\cos 3\theta$ in terms of $\cos\theta$

(2) Verify the identity: $\frac{\sec^2\theta}{1-\tan^2\theta} = \sec 2\theta$

Even More Exercises:

(1) Find all solutions to $\cos 2x + 2\cos^2 x = 0$

(2) Find all solutions to $2\sin 2x = \sqrt{3}$

Other Trigonometric Identities

- Power-Reducing Identities (Reduction Formulas)

  $\sin^2\theta = \frac{1-\cos 2\theta}{2}$ $\quad$ $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\quad$ $\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$

- Product-to-Sum Identities

  $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta) + \cos(\alpha-\beta))$ $\quad$ $\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

  $\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha-\beta) - \cos(\alpha+\beta))$

- Sum-to-Product Identities

  $\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$ $\quad$ $\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$

  $\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$ $\quad$ $\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$