Section 8.6 Lecture Notes

Complex Numbers Review

Definition: The *imaginary unit* i is defined as a square root of -1. In other words, $\sqrt{-1} = i$.

Note: The property $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ is *not* true when both a and b are negative.

Examples: Simplify the following expressions using i.

(1) $\sqrt{-16}$

(2) $\frac{2 \pm \sqrt{-24}}{2}$

(3) $\sqrt{-3} \cdot \sqrt{-3}$

(4) $\sqrt{-2} \cdot \sqrt{-8}$

(5) $(-2 + 3i) + (4 - 6i)$

(6) $(5 - 7i) - (8 + 3i)$

(7) $(-5 + i)(7 - 9i)$

(8) $(-3 + 2i)^2$

(9) $\frac{17}{4+i}$

(10) $\frac{3+2i}{5-i}$

(11) i^{19}

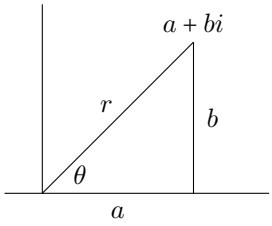
(12) i^8

New Material

Definition: Any complex number can be expressed in standard form $a + bi$, where a and b are real numbers. a is called the *real part* and b is called the *imaginary part*.

Graphing Complex Numbers: Just like real numbers can be graphed on a number line, complex numbers can be graphed on a *complex plane*, where the horizontal axis is the *real axis* and the vertical axis is the *imaginary axis*.

Brainstorm: What equations relate θ, r, a, and b?



Definitions: Given a complex number $a + bi$,

- The *trigonometric form* is $r(\cos\theta + i\sin\theta)$, where $a = r\cos\theta$ and $b = r\sin\theta$.
- $r = \sqrt{a^2 + b^2}$ is called the *modulus* of the number
- θ is called the *argument*.

Examples: Find the trigonometric form for each complex number.

(1) $1 + i$

(2) $-1 - i\sqrt{3}$

Examples: Rewrite the trigonometric form as $a + bi$, where a and b are real numbers.

(1) $4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

(2) $\sqrt{3}(\cos 150^\circ + i\sin 150^\circ)$

Brainstorm: What happens when you multiply two trigonometric forms $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ by each other? What about when you divide them?

Example: Find the product and quotient of $4(\cos 45^\circ + i \sin 45^\circ)$ and $2(\cos 135^\circ + i \sin 135^\circ)$.

Brainstorm: What happens when you raise a trigonometric form $z = r(\cos\theta + i \sin\theta)$ to a positive power (i.e. what is z^n)?