Final Exam Practice Questions

Note: This document is split up based on major mathematical themes covered throughout the course. A topic that is not on this practice final may still show up on the actual final exam if it was covered in class.

Exponential and Logarithmic Functions

(1) The half-life of a radioactive substance is 9 years. Initially a sample has 20 grams. How many grams remain after 8 years?

(2) Initially, 200 bacteria are present in a colony. Eight hours later there are 500. What is the population two hours after the start?

(3) Initially a bank account that is compounded continously has \$4,000. 10 years later it has \$12,000. Find the amount after 5 years.

(4) Graph $y = 1 - \ln(2 - x)$ labelling all asymptotes. Then find the domain and range.

(5) Graph $y = \ln(-3 - x) - 2$ and answer the following questions:

- a) What is the equation of the asymptote?
- b) What is the domain?
- c) What is the range?

(6) Combine into a single logarithm: $\ln(x + y) - \ln(xy) - \frac{1}{2}\ln(x - 1)$

(7) Combine into a single logarithm: $2\ln\left(\frac{1}{x}\right) + 3\ln(x + 1)$

(8) Combine into a single logarithm (with no coefficient): $\ln(x^2 + y^2) + 2\ln(y^3)$

(9) Write as a sum/difference of multiples of the simplest possible logarithms: $\log_2 \sqrt{\frac{xy}{(y - x)^3}}$

(10) Write as a sum/difference of multiples of the simplest possible logarithms: $\log_b \left(\frac{\sqrt[3]{x}}{x + 1}\right)$

(11) Write as sum/difference of **multiples** of the simplest possible logarithms: $\log_b \left(\frac{\sqrt[4]{2y}}{y^3 + a}\right)$

(12) Simplify $\log_9 \left(\frac{1}{3}\right)$

(13) Simplify $\log_2 \left(\frac{1}{\sqrt{2}}\right)$

(14) Simplify $\ln(e\sqrt{e})$

(15) Express using natural logarithms: $\log_{10}(14)$

(16) Solve $e^{3x} = 8$

(17) Solve $e^{2x+4} = 9$

(18) Solve $\ln(x+1) + \ln(x+2) = \ln(6)$

(19) Solve $\ln(x+3) + \ln(x+4) = \ln(2)$

(20) Solve $\ln x = 1 + \ln(x-1)$

(21) Solve $\ln(2x) = 1 + \ln(x-1)$

(22) Solve $\ln(x) + \ln(x-1) = \ln 2$

(23) Solve $(3^{2x})^5 = 10$

(24) Solve $3^x = 2^{x+1}$

(25) Solve $3^{2x-3} = \sqrt{3}$

Trigonometric Functions and Inverse Trigonometric Functions

(1) Convert 300° to radians.

(2) Find the radian measure of an angle which intercepts a 10 inch arc on a circle of radius 36 inches.

(3) Find $\cot\left(\frac{11\pi}{3}\right)$

(4) If $\cos(\theta) = -\frac{4}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find $\sin\theta$.

(5) If $\cos\theta = \frac{4}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$, find $\sin\theta$

(6) Suppose $\sin(\theta) = -\frac{1}{4}$ and $\pi < \theta < \frac{3\pi}{2}$. Find $\cos(\theta)$

(7) The side opposite angle θ of a right triangle is 6. The hypotenuse length is x^2, find $\tan(2\pi - \theta)$ and $\tan\left(\frac{\pi}{2} - \theta\right)$.

(8) Graph $y = -2\sin\left(\frac{x}{4} - \frac{\pi}{2}\right)$ over one period. Label the x-values of the four parts.

(9) Graph $y = \cos(\pi x - \pi)$ **over one period**.

(10) Graph $y = \tan\left(-\frac{x}{2} + \pi\right)$ **over one period**

(11) Find $\tan\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

(12) Find $\sin^{-1}\left(\sin\left(\frac{5\pi}{7}\right)\right)$

Trigonometric Identities and Equations

(1) Simplify $\dfrac{\sin(\theta)}{1+\cos(\theta)} + \dfrac{1+\cos(\theta)}{\sin(\theta)}$

(2) Simplify $\tan(t)(\cos t + \cos(-t))$

(3) Simplify $\dfrac{\cot^2 t(\sec^2 t - 1)}{\sec^2 t - \tan^2 t + 1}$

(4) Prove $\csc t = \sin t + \cot t \cos t$

(5) Prove $\dfrac{\sec^2\theta - \tan^2\theta}{1+\cot^2\theta} = \sin^2\theta$

(6) Simplify $\sin(x+y)\cos x - \cos(x+y)\sin x$

(7) Simplify $\dfrac{\tan(\pi/5) - \tan(\pi/30)}{1+\tan(\pi/5)\tan(\pi/30)}$

(8) If $\tan(s) = 2$ and $\tan(t) = 3$, find $\tan(s+t)$.

(9) If $\sin\theta = 2x$ and $\dfrac{\pi}{2} < \theta < \pi$, find $\sin(2\theta)$

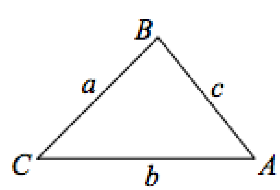
(10) If $\sin\theta = 2x$ and $\dfrac{\pi}{2} < \theta < \pi$, find $\sin\left(\dfrac{\theta}{2}\right)$

(11) Find all solutions to $2\cos^2 x - \sin x - 1 = 0$

(12) Find all solutions to $2\cos^2 x - 5\cos x = -2$

(13) Find all solutions to $\cos^2 x + \sin x + 1 = 0$

(14) Consider the following triangle



- (a) If $a = 3$, $b = 1$, and $c = 3$, find $\angle B$.
- (b) If $a = 3$, $\angle C = 60°$, and $\angle A = 50°$, find b.
- (c) If $a = 3$, $b = 2$, and $\angle B = 30°$, solve for $\angle C$.
- (d) If $a = 3$, $c = 2$, and $\angle C = 60°$, find b.

(15) The two legs of a right triangle are 2 and 3. What is the cosine of the smallest angle? (Hint: the smallest angle is opposite the smallest side)

Polar Equations, Trigonometric Forms of Complex Numbers, and Systems of Equations

(1) Convert the polar equation to a rectangular equation: $2\sin\theta - 3\cos\theta = r$

(2) Convert the rectangular equation to a polar equation: $y = x^2$

(3) Convert $\left(2, \dfrac{\pi}{4}\right)$ to rectangular coordinates. Convert (π, π) to polar coordinates.

(4) Convert the polar equation to a rectangular equation: $r^2 = \cos(2\theta)$

(5) Convert the rectangular equation to a polar equation: $x^2 - y^2 = 1$

(6) Write $\cos 150^\circ + i\sin 150^\circ$ in standard form

(7) Write $5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in standard form

(8) Write $-\sqrt{2} - i\sqrt{2}$ in trigonometric form

(9) Write $\sqrt{3} + i$ in trigonometric form

(10) Write $-1 + i\sqrt{3}$ in trigonometric form

(11) Compute the operation and leave your answer in trigonometric form:
$6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(12) Compute the operation, then convert your answer to standard form:
$5(\cos 90^\circ + i\sin 90^\circ) \div 2(\cos 30^\circ + i\sin 30^\circ)$

(13) Compute the operation and leave your answer in trigonometric form:
$9(\cos 45^\circ + i\sin 45^\circ) \cdot 3(\cos 15^\circ + i\sin 15^\circ)$

(14) Compute the operation, then convert your answer to standard form:
$\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) \cdot \left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

(15) Compute the operation, then convert your answer to standard form:
$\left(3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^4$

(16) Compute the operation, then convert your answer to standard form:
$(2(\cos 90^\circ + i\sin 90^\circ))^5$

(17) Solve: $\begin{array}{l} 4x = 3y + 17 \\ x = -2y - 4 \end{array}$

(18) Solve: $\begin{array}{l} 6x + 3y = 30 \\ 2x + 3y = 18 \end{array}$

(19) Solve: $\begin{array}{l} x - 3y = 4 \\ -2x + 6y = 1 \end{array}$

(20) Solve: $\begin{array}{l} 2x + y - 2z = -1 \\ 3x - 3y - z - z = 5 \\ x - 2y + 3z = 6 \end{array}$

(21) Solve:
$$\begin{aligned} x + y + z &= 2 \\ y - 3z &= 1 \\ 2x + y + 5z &= 0 \end{aligned}$$

(22) Solve:
$$\begin{aligned} 2x + y - 3z &= 0 \\ 4x + 2y - 6z &= 0 \\ x - y + z &= 0 \end{aligned}$$