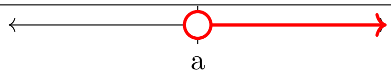
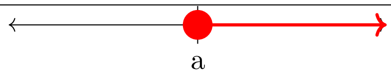
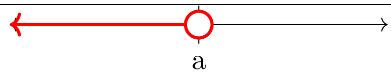
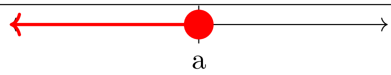
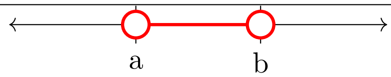
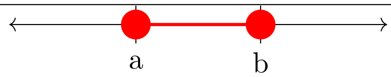
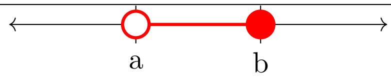
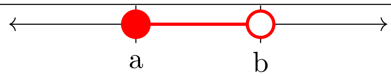
Reference Sheet

Interval Notation

Inequality	Number Line	Interval Notation
$x > a$		(a, ∞)
$x \geq a$		$[a, \infty)$
$x < a$		$(-\infty, a)$
$x \leq a$		$(-\infty, a]$
$a < x < b$		(a, b)
$a \leq x \leq b$		$[a, b]$
$a < x \leq b$		$(a, b]$
$a \leq x < b$		$[a, b)$

Properties of Exponents

- $b^n b^m = b^{n+m}$
- $\frac{b^n}{b^m} = b^{n-m}, b \neq 0$
- $b^0 = 1, b \neq 0$
- $(b^n)^m = b^{nm}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
- $b^{-n} = \dfrac{1}{b^n}, b \neq 0$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ for $a, b \neq 0$
- $\sqrt[n]{x} = y \Rightarrow y^n = x$
- $\sqrt[n]{x} = x^{1/n}$
- $x^{m/n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$

Exponential and Logarithmic Functions

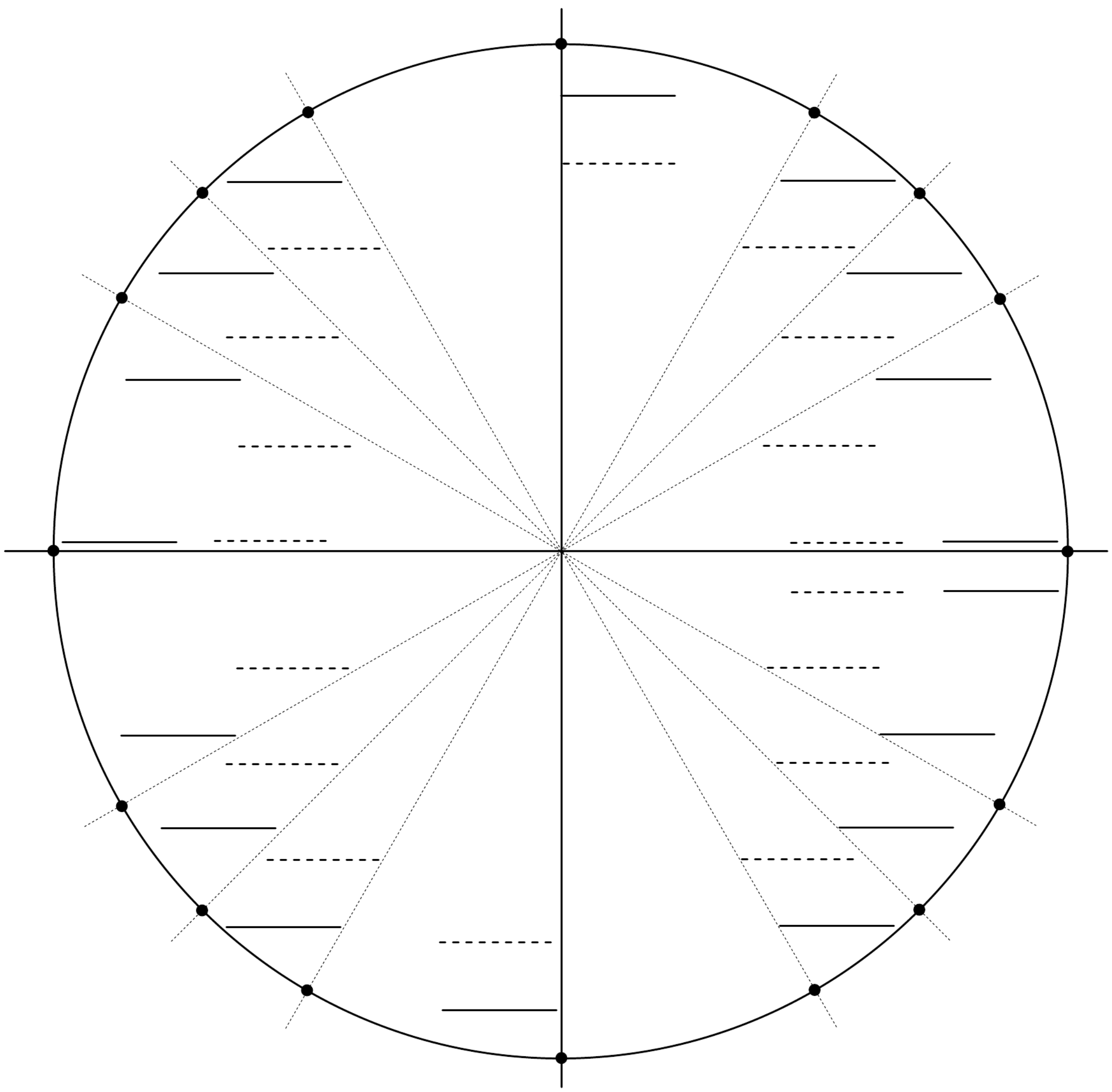
- An exponential function is a function of the form $f(x) = Ca^x$, $a > 0, a \neq 1$
- The formula for **compound interest** (compounded n times per year) is $A = P\left(1 + \frac{r}{n}\right)^{nt}$
- The formula for **continuously compounded interest** is $A = Pe^{rt}$
- The **half-life** exponential formula is $A(t) = C\left(\frac{1}{2}\right)^{t/k}$
- A logarithmic function is a function of the form $f(x) = \log_b x$, with $x > 0$
- $\log_b x = y \Leftrightarrow x = b^y$
- $\log_e x = \ln x$
- $\log_{10} x = \log x$
- $\log_b b^x = x$
- $b^{\log_b x} = x$, for $x > 0$.
- $\log_b 1 = 0$
- $\log_b x + \log_b y = \log_b xy$
- $\log_b x - \log_b y = \log_b \frac{x}{y}$
- $\log_b(x^r) = r \log_b x$
- $\log_b x = \frac{\log_a x}{\log_a b}$

Trigonometric Functions

- Conversion between degrees and radians: $180° = \pi$ radians.
- Arc length formula: $\theta = \frac{s}{r}$, where θ is in radians.
- Pythagorean theorem: $a^2 + b^2 = c^2$, where c is the length of the hypotenuse.
- $\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
- $\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$
- $\csc(\theta) = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$
- $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$
- $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$

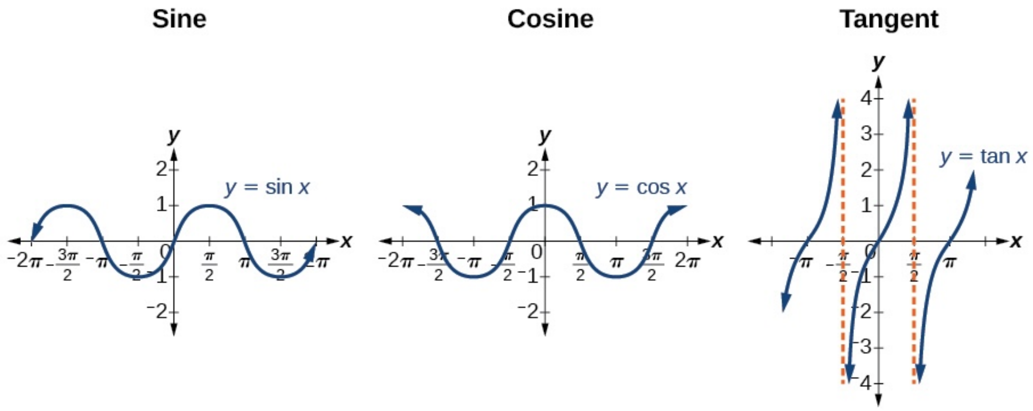
Unit Circle

NOTE: You cannot fill in this unit circle ahead of the exam.



Graphing Trigonometric Functions

General graphs:



Steps for graphing $f(x) = \pm A\sin(Bx \pm C) \pm D$, $f(x) = \pm A\cos(Bx \pm C) \pm D$, and $f(x) = \pm A\tan(Bx \pm C) \pm D$:

- Factor out B, if necessary.
- Amplitude for sine and cosine is determined by A. There is no amplitude for tangent. (A negative in front of the A would indicate a reflection across the x axis.)
- Period for sine and cosine is $\frac{2\pi}{B}$. Period for tangent is $\frac{\pi}{B}$
- Phase shift (horizontal shift) is determined by $\frac{C}{B}$. If there is a minus, it shifts to the right. If there is a plus, it shifts to the left.
- Vertical shift is determined by D. If there is a plus, it shifts up. If there is a minus, it shifts down.
- Determine the start point and end point for one period using the information above, then graph the general shape. Label relevant positions on the x and y axes.

Inverse Trigonometric Functions

- $\sin^{-1}(x) = \arcsin x$
- $\cos^{-1}(x) = \arccos x$
- $\tan^{-1}(x) = \arctan x$
- The output of $\sin^{-1}(x)$ is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- The output of $\cos^{-1}(x)$ is restricted to $[0, \pi]$
- The output of $\tan^{-1}(x)$ is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\sin^{-1}(\sin\theta) = \theta$ if $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- $\cos^{-1}(\cos\theta) = \theta$ if $\theta \in [0, \pi]$
- $\tan^{-1}(\tan\theta) = \theta$ if $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- $\sin\left(\sin^{-1} x\right) = x$ if $x \in [-1, 1]$
- $\cos\left(\cos^{-1} x\right) = x$ if $x \in [-1, 1]$
- $\tan\left(\tan^{-1} x\right) = x$ for all x

Reciprocal Identities

- $\sin\theta = \frac{1}{\csc\theta}$
- $\cos\theta = \frac{1}{\sec\theta}$
- $\tan\theta = \frac{1}{\cot\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$

Quotient Identities

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Period Identities/Theorems: For integers n

- $\sin(\theta \pm n\pi) = \begin{cases} \sin\theta, & \text{if } n \text{ is even} \\ -\sin\theta, & \text{if } n \text{ is odd} \end{cases}$
- $\cos(\theta \pm n\pi) = \begin{cases} \cos\theta, & \text{if } n \text{ is even} \\ -\cos\theta, & \text{if } n \text{ is odd} \end{cases}$
- $\tan(\theta \pm n\pi) = \tan\theta$, for all n

Co-function Identities:

- $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
- $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$
- $\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$
- $\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$
- $\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Sum and Difference Identities (Trigonometric Addition Formulas):

- $\sin(s+t) = \sin s\cos t + \cos s\sin t$
- $\sin(s-t) = \sin s\cos t - \cos s\sin t$
- $\cos(s+t) = \cos s\cos t - \sin s\sin t$
- $\cos(s-t) = \cos s\cos t + \sin s\sin t$
- $\tan(s+t) = \dfrac{\tan s + \tan t}{1 - \tan s\tan t}$
- $\tan(s-t) = \dfrac{\tan s - \tan t}{1 + \tan s\tan t}$

Double Angle Identities:

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  $= 2\cos^2\theta - 1$
  $= 1 - 2\sin^2\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Half-Angle Identities:

- $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
- $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
- $\tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\theta}{1+\cos\theta}$