Reference Sheet

Properties of Real Numbers

+ indicates the property is for addition. × indicates the property is for multiplication.

- Commutative property (+): $x + y = y + x$
- Commutative property (×): $xy = yx$
- Associative property (+): $(x + y) + z = x + (y + z)$
- Associative property (×): $(xy)z = x(yz)$
- The identity property (+): $x + 0 = x$
- The identity property (×): $1 \cdot x = x$
- The inverse property (+): $a + (-a) = 0$
- The inverse property (×): $a \cdot \frac{1}{a} = 1$
- The distributive property: $x(y + z) = xy + xz$

Fraction & Rational Function Arithmetic

- $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$
- $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$
- To add and subtract fractions and rational functions, you need a common denominator.
  $\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$ and $\dfrac{a}{c} - \dfrac{b}{c} = \dfrac{a-b}{c}$

Absolute Values (for $a \geq 0$)

- If $|x| = a$, then $x = a$ or $x = -a$
- If $|x| < a$, then $-a < x < a$
- If $|x| > a$, then $x < -a$ or $x > a$

Linear Functions

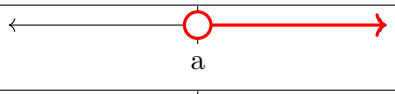
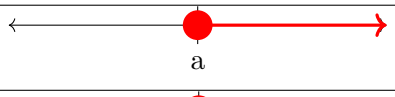
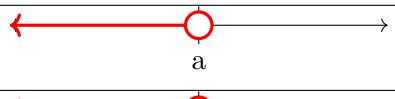
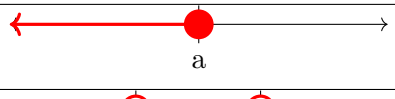
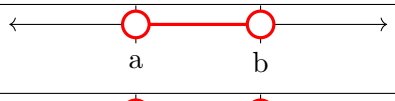
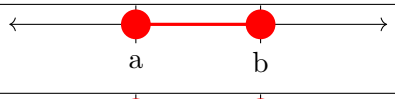
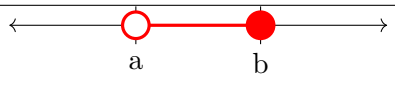
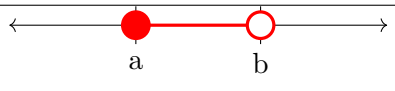
- Standard/General Form: $Ax + By + C = 0$, where A and B aren't both 0
- Slope-Intercept Form: $y = mx + b$, where m is the slope and b is the y-intercept
- Point-Slope Form: $y - y_1 = m(x - x_1)$, where m is the slope and the point (x_1, y_1) is on the line
- $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Parallel lines have the same slope. Perpendicular lines have slopes that are opposite reciprocals of each other.

Properties of Exponents

- $b^n b^m = b^{n+m}$
- $\dfrac{b^n}{b^m} = b^{n-m}$
- $b^0 = 1$
- $(b^n)^m = b^{nm}$
- $(ab)^n = a^n b^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
- $b^{-n} = \dfrac{1}{b^n}$
- $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

Reference Sheet Continued

Inequalities

Inequality	Number Line	Interval Notation
$x > a$		(a, ∞)
$x \geq a$		$[a, \infty)$
$x < a$		$(-\infty, a)$
$x \leq a$		$(-\infty, a]$
$a < x < b$		(a, b)
$a \leq x \leq b$		$[a, b]$
$a < x \leq b$		$(a, b]$
$a \leq x < b$		$[a, b)$

Miscellaneous

- $a \times 10^n$ means move the decimal n places to the right. $a \times 10^{-n}$ indicates that you move the decimal n places to the left.
- To convert from a percentage to a decimal, divide the number by 100 ("per" means "out of" and "cent" means 100). This is equivalent to moving the decimal two places to the left.
- To solve a system of equations, you can:
  - Solve by substitution (solve for one variable and plug it into the other equation)
  - Solve by elimination (multiply the equations by a number to make the coefficients add up, then add or subtract them from each other)
- To graph an inequality, plot the equation of the line (determine if it is dashed or solid), then test any point not on the line to determine which side to shade
- To solve a system of inequalities, graph both inequalities and find where their shaded regions overlap.
- To solve a rational function equation, multiply both sides by a common denominator to clear the fractions. Then solve and check your answers.

Polynomials

- Distributive property: $(a+b)(c+d) = ac + bc + ad + bd$
- The greatest common factor (GCF) is the product of the factors common to all terms in the polynomial.
- Factor by grouping: $a(b+c) + d(b+c) = (a+d)(b+c)$
- To factor $ax^2 + bx + c$ by grouping, find two numbers that multiply to $a \cdot c$ and add to b. Use these two numbers to split up the middle term bx.
- Difference of squares formula: $a^2 - b^2 = (a-b)(a+b)$
- Difference of cubes formula: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- Sum of cubes formula: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Reference Sheet Continued

Polynomials Continued

- Solving a polynomial equation: Get everything to one side of the equation, factor the non-zero side of the equation, then set each factor equal to 0 and solve.
- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- To divide polynomials, use either long division or synthetic division (if synthetic division applies).

Select Examples:

(1) Polynomial Long Division:

$$\begin{array}{r} 3x^2 + 3x - 1 \\ 3x+1 \overline{)\ 9x^3 + 12x^2 \quad\quad + 3} \\ \underline{-9x^3 - 3x^2} \\ 9x^2 \\ \underline{-9x^2 - 3x} \\ -3x + 3 \\ \underline{3x + 1} \\ 4 \end{array}$$

(2) Rational function addition and simplification:

$$\begin{aligned} \frac{2x}{x^2-1} + \frac{1}{x^2+x} &= \frac{2x}{(x+1)(x-1)} + \frac{1}{x(x+1)} \\ &= \frac{2x}{(x+1)(x-1)} \cdot \frac{x}{x} + \frac{1}{x(x+1)} \cdot \frac{x-1}{x-1} \\ &= \frac{2x^2}{x(x+1)(x-1)} + \frac{x-1}{x(x+1)(x-1)} \\ &= \frac{2x^2 + x - 1}{x(x+1)(x-1)} \\ &= \frac{(2x-1)\cancel{(x+1)}}{x\cancel{(x+1)}(x-1)} \\ &= \frac{2x-1}{x(x-1)} \end{aligned}$$

Radicals & Rational Exponents

- $\sqrt[n]{x} = y \Rightarrow y^n = x$
- $\sqrt[n]{x} = x^{1/n}$
- $x^{m/n} = \left(\sqrt[n]{x}\right)^m = \sqrt[n]{x^m}$
- Solving a radical equation: Isolate the term with the radical (root sign), then raise both sides to the power that matches the radical. Combine like terms and solve. Make sure to check your answers to see if they are valid.